Reliability approach on impacted composite materials for railways

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RÉSUMÉ. This paper deals with a reliability approach applied on composite plates which should be used in railway structures under low velocity impact loading.
Impacted composite plates in bending configuration are considered as an application of the reliability approach to composite materials. Mass of projectile, height of fall and distance between supports are the three variables considered. A limit state function G is defined by the impact force compared to a critical one. Using reliability approach, the reliability index β allows to estimate the probability of failure P_f. It is then assessed for several values of the critical force using Monte Carlo direct method and First Order Reliability Method (FORM) approximated method. For FORM method, genetic algorithms are investigated to obtain the best reliability index β. Results obtained by the approximated FORM method are finally discussed and compared to Monte Carlo simulations considered as a reference .

KEYWORDS: impact, damage, reliability, genetic algorithm.

1. Introduction

Fatigue is known to be responsible for the majority of failures of structural components in transportation applications but impact loading is also critical due to damage not always visible (Ballère *& al.*, 2009). The main subject of this research is to develop a reliability approach (Méalier & al., 2010; Lemaire, 2008) for analyzing a composite plate under low velocity impact loading when variabilities are taken into account . Applying such a reliability approach on composite materials is not so usual in literature.

Both experimental and numerical aspects are performed in the present work. Experimental investigations concern the impact tests on composite plates using experimental design technique while numerical ones are related to mechanical reliability models based on experimental results to predict the probability of failure taking into account variabilities. Impact tests have been carried out to understand the main damage mechanisms and to determine the different mechanical responses of the impacted structure: the contact force between the striker and the sample, the contact duration and the out of plane displacement. Previous mechanical responses have been represented using the Response Surface Methodology (RSM) established from the experimental design. Each investigated response is then modeled by an empirical polynomial for which the coefficients are calculated from the chosen experimental design (Doehlert, 1970, 1972). For reliability analysis, the projectile mass (m), the height fall (h) and the span of the simply supported composite plate (p) are considered as mechanical uncertain input variables. The maximum value of the contact force, denoted by $F_{contact}$, is analysed as the mechanical output response variability thanks to reliability tools so that the reliability index β and the failure probability level P_f could be achieved. Then, direct Monte Carlo method and approximated First Order Reliability Method (FORM) are especially investigated. The reliability index β , needed for numerical FORM application is determined using Genetic Algorithm (GA) (Cantú-Paz, 2000) coupled with Deterministic Algorithm (DA), an original alternative (DA) mainly used in the most of current reliability tools. Results obtained by the approximated FORM method are finally discussed and compared to Monte Carlo reference simulations before concluding.

2. Experimental stage

2.1. Impact device

The impact tests have been performed using a dropping mass set-up designed in our laboratory to simulate accidental falls on a structure, Figure 1. The contact force history between the striker and the composite specimen is measured by a piezoelectric sensor clamped to the mass. A first laser sensor placed just underneath the center of the specimen provides the out-of-plane displacement history. Such a sensor allows measurement without contact. It is particularly adapted to dynamic

measurement. A second laser sensor measures the striker displacement versus time so that the velocity of the dropping mass can easily be assessed.

2.2. Composite samples and striker

The plate used in this study is made of an eight layers carbon/thermoset laminated composite. It was elaborated by hand lay-up technique and was cured under vacuum. Samples were cut from a large plate for which : i) the total thickness is close to 4.2mm and ii) the fiber volume fraction is estimated to 56%.
Each sample is simply supported on two metallic supports fixed on a solid mass and impacted on its center, Figure 1.
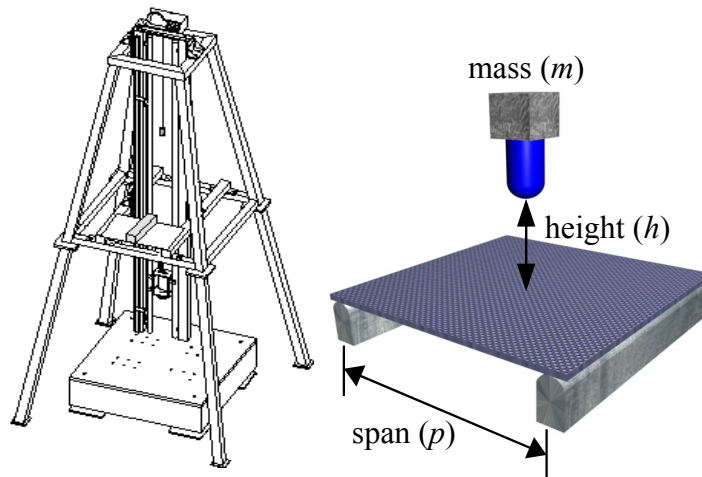The stricker is hemispherical of diameter 16 mm. It is supposed to be indeformable.

Fig. 1. Drop tower and composite sample.

2.3. Impact tests

Three main parameters have been chosen to qualify a test : mass and velocity of the striker and the span. The width of the plate is defined such as the ratio 'width/span' remains a constant value (about 3.00) so that results could be compared more easily.
For each test the different values for all the parameters are suggested by an experimental design (Droesbeker, 1997). Doelhert experimental design based on simplex method is adopted (Doehlert, 1970, 1972) due to its main properties: i) the experimental points are uniformly distributed in the experimental design space ii) new variable can be easily added iii) this is a second order design space which permits to elaborate surface response on the mathematical form given in Eq. (1).

$$Y = a_0 + \sum_{i=1}^{n} a_i . x_i + \sum_{i=1}^{n} a_{ii} . x_i . x_i + \sum_{i=1}^{n} \sum_{j>i}^{n} a_{ij} . x_i . x_j \qquad (1)$$

Setting up an experimental design method involves several steps. Firstly, the nature and limit values of the variables must be fixed. The degree of the polynomial must then be chosen. Thereafter, the matrix giving all the values for the parameters can be established according to strict rules (Droesbeker, 1997). Moreover, in order to make easier the determination of the relative influence of the different parameters, the variables are expressed in centred, reduced co-ordinates. Their amplitude of variation is therefore standardized into the range [-1,+1]. Codification is detailed in Table 1.

Table 1 : Impact test parameters in physical and centred reduced co-ordinates.

Variable	Name	Level +1	Level 0	Level -1
mass (kg)	x_1	3,50	2,75	2
height (m)	x_2	1,0	0,6	0,2
span (mm)	x_3	480	305	130

Using Doelhert theory, the experimental matrix is given by Table 2.

Table 2 : Test matrix using Doelhert theory.

N°	1	2	3	4	5	6	7	8	9	10	11	12	13
X_1	0	1	-1	0.5	-0.5	0.5	-0.5	0.5	-0.5	0.5	0	-0.5	0
X_2	0	0	0	0.87	-0.87	-0.87	0.87	0.29	-0.29	-0.29	0.58	0.29	-0.58
X_3	0	0	0	0	0	0	0	0.82	-0.82	-0.82	-0.82	0.82	0.82

We can notice that only 13 tests are needed to obtain the empirical polynomial including the influence of the coupling between the different variables (X_i). A last interesting point of the Dolhert approach is that all the variables can not have the same number of level. For instance, x_1 can take five different values (-5, -1, 0, 1, 5) while x_2 can change 7 times (-0.87, -0.58, -0.29, 0, 0.29, 0.58, 0.87). Advantages are: i) to increase the polynomial approximation accuracy according to particular variables, ii) to limit the number of device settings when discrete parameter is involved (the mass for instance).

Then, polynomial coefficients, $\hat{a}$, are estimated using a multilinear regression, Eq. (2).

$$\hat{a} = (x^t x)^{-1} x^t y \qquad (2)$$

with $\hat{a} = (a_0 \, a_1 \, a_2 \, a_3 \, a_{12} \, a_{13} \, a_{23} \, a_{11} \, a_{22} \, a_{33})$, (x) a matrix for which each row corresponds to a realization $(1 \quad x_1 \quad x_2 \quad x_3 \quad x_1 x_2 \quad x_1 x_3 \quad x_2 x_3 \quad x_1^2 \quad x_2^2 \quad x_3^2)$, and (y) the output column matrix.

The quality of the polynomial (Eq. (1)) has to be checked using the variance analysis (Saporta, 1990). It consists in verifying that the approximated response variation, $F_{contact}$, is really involved by variation of parameters X_1, X_2 and X_3 and not due to experimental noise. Moreover, the total error coming from the choice of the experimental design can be established using Eq. (3) before doing any tests.

$$e=\left(x^t x\right)^{-1} \tag{3}$$

An example of e considering only the variable X_1, X_2, is illustrated in Fig. 2. It can be noticed that the accuracy is lower around the corners of the domain.
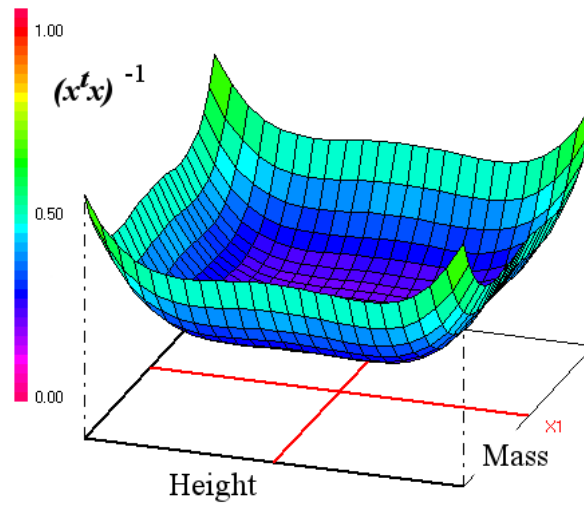

Fig. 2. Error due to the type of the investigated experimental design.

Although the analysis of the impact test responses is not the main objective of this work, an example of a registered curve Force versus Time is given in Fig. 3. It represents a mechanical response as a superposition of a quasi-static behaviour and a vibratory part which mainly reveals the striker, the plate and metallic supports eigen modes participation. More details about mechanical behaviour of impacted composites are available in literature (Guillaumat, 2000, 2005).

$F_{contact}$ will correspond to the maximum measured value, that is to say *569 N* in the present case.

The damage, induced by the impact, is mainly represented by longitudinal localized cracks under the contact zone between the sample and the striker and delamination always inside the cracked zone, Fig. 4.
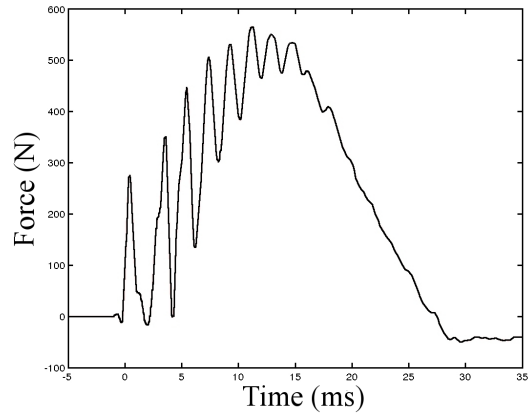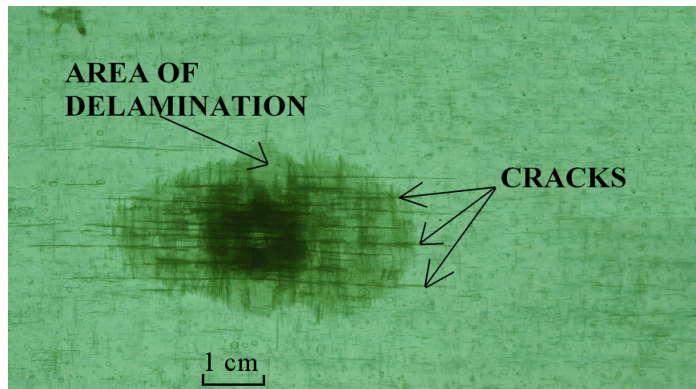
Fig. 3. Typical force versus time curve.

Fig. 4. Delamination and matrix cracks after impact.

3. Response modelisation

Based on the previous experimental design, the contact force $F_{contact}$ is modeled by a response surface avoiding heavy calculations with (FE) or numerous experimental tests to assess the variability of the response and perform future reliability analysis.

The $F_{contact}$ polynomial estimated by the Eq.2 from the experimental results according to the Doelhert's experimental design (Doehlert, 1970, 1972), is expressed by Eq. (4).

$$F_{contact} = 1272 + 171\,x_1 + 485\,x_2 - 516\,x_3 - 20\,x_1 x_2 + 71\,x_1 x_3 - 79\,x_2 x_3 \tag{4}$$
$$- 97\,x_1^2 - 49\,x_2^2 + 262\,x_3^2$$

Using Eq. (4), $F_{contact}$ response can be then simply evaluated with minimum cost; only 13 tests are needed to establish this polynomial. A plot of this response is illustrated on Fig. 5 when the mass parameter is fixed to its mean value while the two other ones vary.
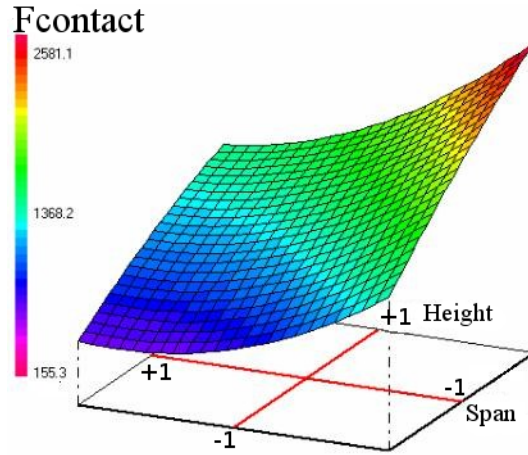

Fig. 5. Response surface of the contact force during impact.

According to this plot, the force evolution seems to be physically acceptable; the span decreases (coming from +1 upto -1) the force increases because of the increase of the sample stiffness. Moreover, a slightly nonlinear shape can be noticed for the response surface.

4. Reliability approach

4.1. Performance function

The reliability approach begins by the definition of the physical mechanism responsible of the structure failure. Then, this mechanism is translated into terms of a performance function, denoted G, Eq. (5), which classically compares the stress to the strength of the investigated phenomenon (Lemaire, 1997). In this study, the maximum contact force $F_{contact}$ (stress) is compared to an ultimate force $F_{ultimate}$ (strength) such as:

$$G = F_{ultimate} - F_{contact} \tag{5}$$

Obviously, the stress and strength can depend on several variables.

G equal to 0 corresponds to the so-called 'limit state function' (Eq. (6)), separating the failure and safety domains as illustrated in Fig. 6.

$$G = F_{ultimate} - F_{contact} = 0 \tag{6}$$

The 'safety domain' (pale area) defined by $G>0$ and the "failure domain" (dark area) by $G<0$ are also illustrated in Fig. 6.
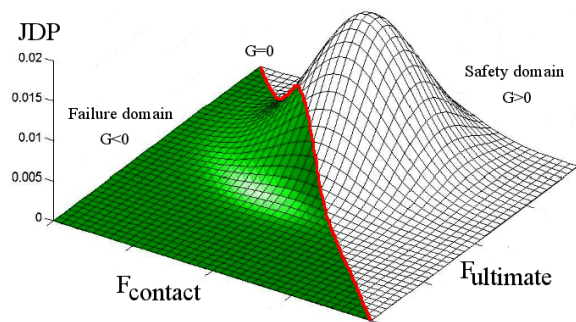

Fig. 6. Joint Density of Probability (JDP)

The next step consists in representing the distribution of uncertain quantities. The mass *(m)*, the height *(h)*, and the span *(p)* are such quantities in this study. They will be described by a normal distribution. This choice doesn't result from any measurements but it is only an assumption coming from the fact that generally physical data is well described by such a distribution.

It is quite obvious that the resulting level of probability can be largely influenced by the nature of the input variable distribution. The proof is given in Fig. 7 where two different statistical distributions (Gaussian and uniform) were used to compare their influences on the probability level. Monte Carlo method (see bellow) was applied on Eq. 6 using Eq. 4 for the $F_{contact}$ to obtain these probabilities.
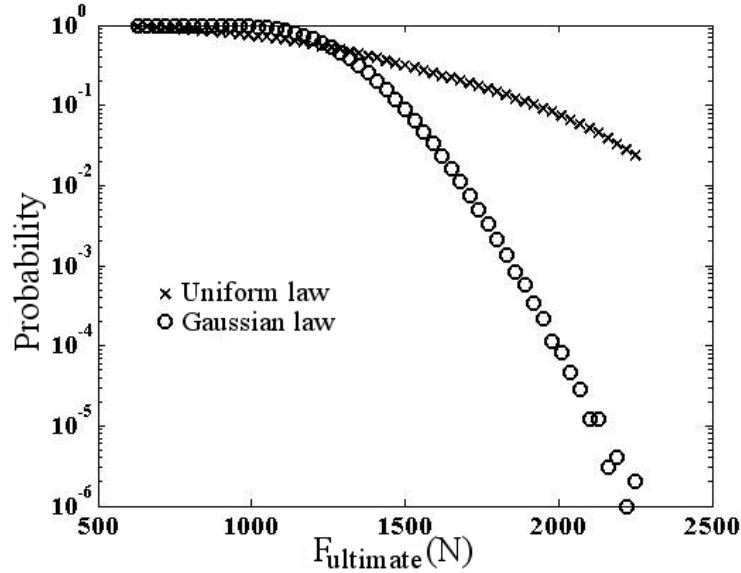
Fig. 7. Statistical distributions influence on probability levels

A great variation between the two situations can be noted. The Gaussian law provides a lower probability level because the values are more concentrated around the mean compared to the uniform law. Thus, the frequency of a value far from the mean is lower in a Gaussian case that implies a lower probability level. This simple example highlights the large influence of the distribution choice which has been established using mechanical tests.

4.2. Failure probability assessment

Classically, several methods are available to perform reliability analysis following the main objective: to estimate the failure probability or a reliability index. To this purpose, the following methods are often used: Monte-Carlo, the derivative Monte Carlo method such as Monte Carlo by importance sampling (Lemaire, 2008), and the others approximated methods such as First Order Reliability Method (FORM) and Second Order Reliability Method (SORM).

In Monte-Carlo approach, all the variables are randomly sampled according to their statistical distribution in order to create a random vector. For each trial (a set of data) the G function; Eq. (5), is then evaluated. The next step consists in counting the number of situations giving G negative. Finally, the failure probability P_f is classically estimated by Eq. (7).

$$\tilde{P}_f = \frac{number \quad of \quad situations \quad G \leq 0}{total \quad number \quad of \quad simulations} \tag{7}$$

Using this method, it is possible to establish the error depending on the trials number (n) and the estimated probability level: $\tilde{P}$. It can advantageously be obtained by Shooman (Shooman, 1968) formula, Eq. (8).

$$\% \, error = 200 \sqrt{\frac{1 - \tilde{P}_f}{n \, \tilde{P}_f}} \qquad (8)$$

On the opposite, this method can be very time consuming when limit state function calculation is complex and needs for instance the use of Finite Element Method (FEM).

From this point of view, the use of a surface response like an empirical polynomial to represent the approximation response $F_{contact}$ (*see section 3*) is quite justified. Then, Monte Carlo approach can be easily performed in this work with minimum cost. Results issued from this 'direct method' will be considered later as reference.

On the other hand, the FORM (and SORM) method consists in an analytical approximation of the failure probability by calculating a reliability index β (Hasofer & Lind, 1974). It is necessary, in this case, to formulate the limit state, Eq. (6), in a reduced variable space, usually called standard space, where each variable distribution becomes a so-called normalized reduced Gaussian distribution after isoprobabilistic transformation. In this standard space, the mean becomes zero, the standard deviation becomes unit, X_i become u_i, G becomes H and all the level curves are circular, Fig. 8.
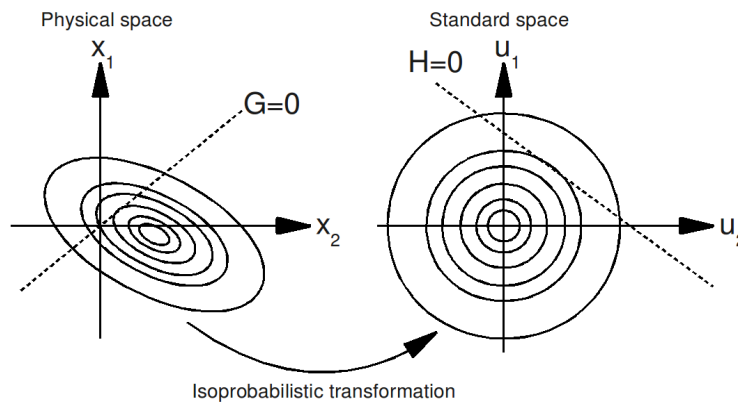

Fig. 8. Isoprobabilistic transformation.

In the case where the statistical distribution in the physical space is a Gaussian, the transformation is simply given by Eq. (9).

$$u_i = \frac{x_i - m_i}{\sigma_i} \qquad \textbf{(9)}$$

m_i and s_i stand for mean and standard deviation of x_i.

In the standard space, the reliability index β represents the shortest distance from the limit state surface (defined by H) to the origin. Optimization algorithms are then needed to determine this index. Deterministic ones are not always the most efficient to find the global minimum depending on the function to minimize. Thus, genetic algorithms are investigated in this work (*see section 4.3*) looking for the best value of this index (global minimum) (Cantú-Paz, 2000).

Then, having β index, the failure probability can be easily deduced by Eq. (10).

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \quad \text{with} \quad \Phi(\beta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta} e^{\frac{-u^2}{2}} \, du \qquad \textbf{(10)}$$

where ϕ stands for the standard normal Cumulative Distribution Function (CDF).

The main assumption in this First Order Reliability Method (FORM) is to consider the limit state as a hyperplane. With FORM method, no error estimation is available contrary to Monte Carlo approach but it is less time consuming.

In general way, coupling several methods for 'reliable' failure probability estimation is preferable in order to obtain the maximum information.

4.3. Obtaining the reliability index β

The failure probability estimation will be as much suitable than β index (needed by FORM approach) issued from optimization algorithms is sufficiently accurate: its determination is of first importance. In the standard space, the optimization problem is formulated as:

$$\beta = \sqrt{\left(\sum_i u_i^2\right)} \qquad (11)$$

Finding u_i which minimize the distance β from the limit state surface to the origin, Fig. 9, and satisfying:

$$H(u_i) = 0 \qquad (12)$$

Function H in Eq. (12) represents the performance function expressed (Eq. (6)) in the standard space. Moreover, by use of optimization algorithms, X_i variables corresponding to β index can be obtained, which is of prime importance for design. Having these coordinates, the statistical influence of every variable on the failure probability level can be estimated.

Let's keep in mind that if we consider only $R = F_{ultimate}$ and $S = F_{contact}$ as two independent Gaussian variables then the β index can be obtained analytically by

Eq. (13). It corresponds to the analytical definition of the smallest distance between the origin and a straight line representing, here, the limit state function.

$$\beta = \frac{\bar{R} - \bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (13)$$

where $\bar{R}$ and $\bar{S}$ respectively denote R and S mean whereas s_R and s_S denote R and S standard deviation.

In this work, the β index is obtained by various ways: either analytically using Eq. (13) with $R = F_{ultimate}$ and $S = F_{contact}$ variables considered in this case as independent Gaussian variables, (*see section 4.2),* or numerically using optimization algorithms. Genetic Algorithms (GA) are especially investigated in this study. In this case, x_1 (mass), x_2 (height) and x_3 (span) are also considered as independent Gaussian variables. It is only an assumption in this work but generally they could be represented by any statistical distribution.

Genetic algorithms (GA) (Cantú-Paz, 2000; Chang, 2006; Cheng, 2007; Deep & Tackur, 2007; He & Sýkora, 2007) are here coupled with deterministic algorithms (DA) to improve global minimum research: the best β value is expected in our case.

Generally known to be time consuming when they are coupled to finite element analysis, GA become powerful when the formulation is quite explicit like in this study and when the limit surface has a complicated shape. GA can reveal themselves more efficient than using only DA, even for simple function. Such a case is illustrated Fig. 9 (Méalier *& al., 2009*) where a paraboloid function is considered. Critical points P_1* or P_2* (feasible exact solutions) are systematically found with GA whereas point P is often obtained using DA; DA are influenced by gradient descent method and does not converge to the right optimum in this case!
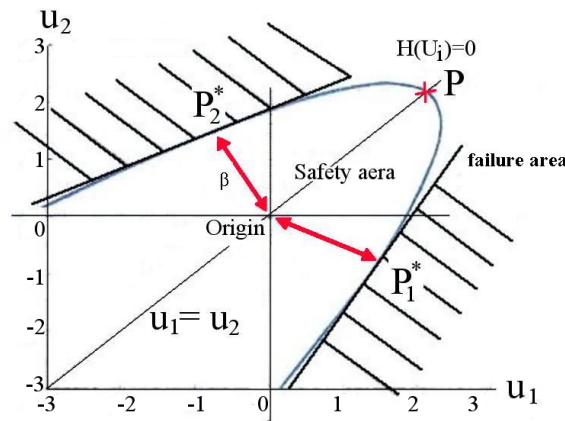

Fig. 9. Paraboloid state limit function.

As a consequence, the estimated failure probability could be very different to the right value because of its high sensitivity to this index (see Equ 10). Fig. 10 gives a

good illustration of this phenomenon. Curves indicate the resulting inaccuracy on the failure probability due to 1%, 2%, 5% and 10% inaccuracy on β index.
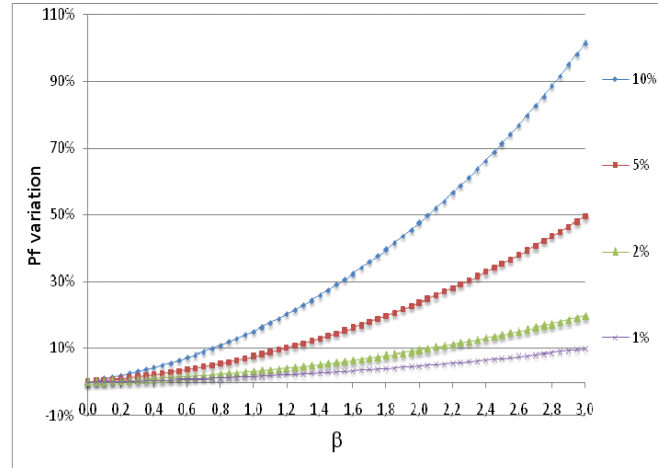

Fig. 10. Influence of β inaccuracy.

Several strategies of research have been investigated for this work to find the better β index. Optimization algorithm GA alone and GA coupled with DA are especially investigated. Based on GA optimization toolbox program in Matlab software (Matlab software), a new program involving 'Islands' strategy GA algorithm (Maeda *& al.,* 2006*)* and coupling the previous GA with classical DA (Haftka & Gürdal, 1993) have been developed. This program, named '*GARelDe*' (as Genetic Algorithm for Reliable Design) is not detailed in this paper. Many tests have been performed using highly nonlinear and multi-modal limit state function from literature. The good performances of GA coupled with DA, using *GARelDe*, are proved in the following investigated *Test 1* and *Test 2*. For these tests, illustrations are given in *Fig. 11* and *Fig. 12,* the data are provided in *Table 3* and the results in *Table 4* and *Table 5*. For each method, 100 simulations have been performed and a percentage of satisfaction is given. The criterion of satisfaction represents the number of times leading to the expected β index value within a fixed error range.

Table 3
Data for *Test1* and *Test 2*.

Test 1
variables and research range: [*x1;x2*] in [-5;5]²
performance function: $G_1(x_{1,}x_2) = e^{(0.4(x_1+2)+6.2)} - e^{(0.3x_2+5)} - 200$

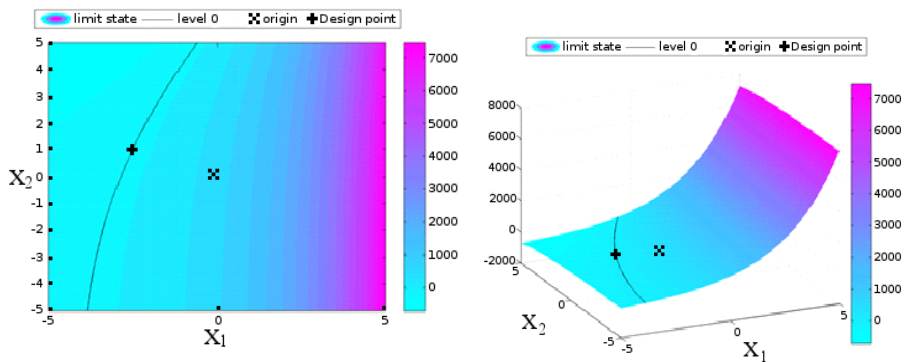| design point theorical value: [-2.5397;0.9454] |
| --- |
| research methods:
- *DAMatool*: Matlab (Matlab software) Deterministic Algorithm
- *GAMatool*: Matlab (Matlab software) Genetic Algorithm
- *GARelDe without coupling*: Present tool without coupling GA and DA
- *GARelDe with coupling*: Present tool with coupling GA and DA |
| population for GA application: *90 individuals* |

Test 2
variables and research range: [x1;x2] in [-10;10]²
performance function: $$G_2(x_1, x_2) = -100 + \sum_i \log(|x_i - 4|) + 50|x_i|$$ $$cos^2(\pi/3\, x_i + 2) + (x_i - 1)^2$$
design point theorical value: [0.989;0.989]
research methods: - *DAMatool*: Matlab (Matlab software) Deterministic Algorithm - *GAMatool*: Matlab (Matlab software) Genetic Algorithm - *GARelDe without coupling*: Present tool without coupling GA and DA - *GARelDe with coupling*: Present tool with coupling GA and DA
population for GA application: *90 individuals*

Fig. 11. Graphical representation of G_1 limit state function (test1).

Table 4
Comparisons of optimization techniques for *Test 1*.

Error on β	±5 %	±10 %
	Satisfaction rate in %	
DAMatool	100	100
GAMatool	5	8
GARelDe (without coupling)	39	48
GARelDe (with coupling)	100	100

Error on β	±2 %
	Satisfaction rate in %
DAMatool	14
GAMatool	54
GARelDe (without coupling)	27
GARelDe (with coupling)	97

For the simple limit state function used in *Test 1*, *DAMatool* and *GARelDe* lead to the same performances. Nevertheless, *DAMatool* remains the most efficient owing to less consuming time compared to *GARelDe with coupling* in this case. For the *Test 2*, *GARelDe with coupling* is the most efficient algorithm.

Fig. 12. Graphical representation of a multi-modal function (test 2)

Finally, a synthesis of results obtained for all tests is given in Table 6. Design point coordinates are also mentioned in this table.

Table 5
Synthesis for all tests.The results obtained by the present *GARelDe* program are compared to Theory, Phimeca (Phimeca software), Elegbede (Elegbede, 2005), Gayton (Gayton & Bourinet, 2003), Borri (Borri & Speranziniones, 1997) and Kiureghian (Kiureghian, 2006) ones.

Performance function	*Method*	x_1	x_2	β
$(x_1-1)^2+(x_2-1)^2$	*Theory*	1	1	1.414
	GARelDe	1	1	1.414
$e^{(0.2x_1+1.4)}-x_2$	*Theory*	-1.680	2.898	3.350
	GARelDe	-1.682	2.897	3.350
	Phimeca	-1.675	2.901	3.350
	Elegbede	-1.688	2.893	3.349
	Gayton	-1.686	2.892	3.348
$-1/2(x_1^2+x_2^2-2x_1x_2)-(x_1+x_2)/\sqrt{2}+3$	*Theory*	-0.765	1.472	1.658
	GARelDe	-0.744	1.482	1.659
	Phimeca	2.121	2.121	3.000
	Borri	-0.752	1.480	1.660
$5-1/2(x_1-0.1)^2-x_2$	*Theory*	-2.741	0.965	2.906
	GARelDe	-2.741	0.965	2.906
	Phimeca	-2.741	0.966	2.906
	Elegbede	-2.742	0.963	2.906
	Kiureghian	-2.741	0.965	2.906
$e^{(0.4(x_1+2)+6.2)}-e^{(0.3x_2+5)}-200$	*Theory*	-2.540	0.945	2.710
	GARelDe	-2.540	0.940	2.710
	Phimeca	-2.540	0.944	2.710
	Elegbede	-2.548	0.924	2.710
$x_1^3+2x_2-5.(x_1-1)^2$	*Theory*	0.617	0.248	0.666
	GARelDe	0.617	0.251	0.666
$x_2^2-(x_1+2)^2-5$	*Theory*	-1.000	2.450	2.646
	GARelDe	-1.001	2.449	2.646

These results confirm the good performance of the association GA (islands strategy , Maeda *& al.*, 2006) and DA in *GARelDe* computer program. For previous reasons, *GARelDe* will be used for present application of impacted composite plate.

5. Simulations and results

All variables distributions are supposed, in this study, to be Gaussian, so completely defined by the mean and the standard deviation.

$F_{ultimate}$ is considered as a variable for this work. Its variation is justified by the $F_{contact}$ mean value of *1271 N,* which is a constant value in Eq. (1). $F_{ultimate}$ *is considered as a* Gaussian distribution, the mean value varies from *600 N* to *2100 N* by increment of *30 N* and the same variance coefficient of *5%* is used for each case. In this way, the probability of failure varies from 1 up to a very small value. For each variable x_1, x_2 and x_3, varying in the range [-1,+1] as suggested by the experimental design (*see section 2.3*), a zero mean value and *0.2* standard deviation are retained. Choosing a small standard deviation avoids to truncate the distribution.

For Monte Carlo approach, the performance function G is directly evaluated by the approximation given in Eq. (4) for $F_{contact}$. A first series of Monte Carlo simulations, denoted *MC1,* is performed. It consists in one million of trials for each variable. For each variable set, function G is evaluated using Eq. (6). The failure probability is finally estimated by Eq. (7). A second series of Monte Carlo simulations, *MC2,* is next performed. For this one, only linear terms in Eq. (4) are considered to evaluate $F_{contact}$ and then G function.

Two ways of calculating the β index are performed for each Monte Carlo calculation: i) an analytical calculation issued from Eq. (13) where $F_{ultimate}$ and $F_{contact}$ are considered as independent Gaussian variables, as explained in section 4.3. Results are denoted *FORM1-Anal* when complete expression given in Eq. (4) is used and *FORM2-Anal* when only first order terms are conserved. ii) a numerical calculation using genetic algorithms where x_1 (mass), x_2 (height) and (span) x_3 are considered as independent Gaussian variables. Results are denoted *FORM1-GA* when complete expression given in Eq. (4) is used and *FORM2-GA* when only first order terms are conserved.

First of all, a comparison between the Monte-Carlo (*MC1)* and FORM (*FORM1-Anal)* methods is shown on Figure 13. The $F_{ultimate}$ mean value, used for the Gaussian distribution with a variance coefficient of *5%,* is mentioned on the curve in X-coordinate.
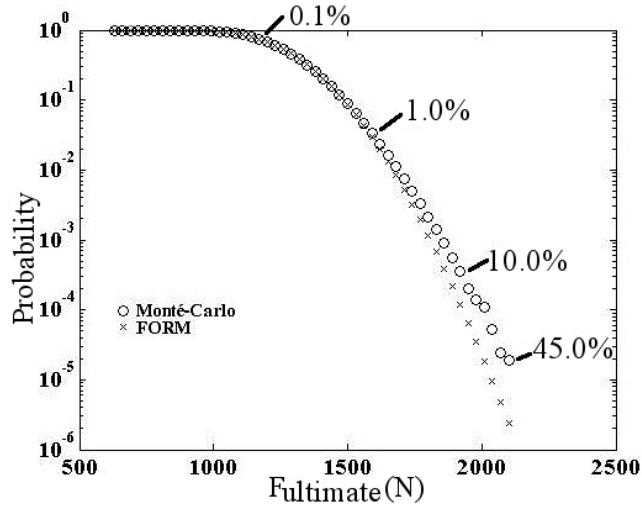
Fig. 13. Probability versus $F_{ultimate}$ estimated by *MC1* and *FORM1-Anal*

A gap between the two simulations is obviously visible beyond $F_{ultimate} = 1500\ N$. The main explanations are due to the fact that i) firstly, the number of trial becomes insufficient as regard to the probability level for the Monte-Carlo technique, ii) secondly, second order polynomial (Eq. (4)) induces a non-Gaussian distribution for $F_{contact}$ (Figure 14). So, an error can be observed in the FORM analytical calculation.

Fig. 14. $F_{contact}$ non-Gaussian distribution

Even if the distribution seems to be not so far from a Gaussian distribution, it has not strictly the right shape. It means that the only knowledge of the mean and the

standard deviation calculated from this distribution are not enough for its complete characterization. Consequently, the FORM method is not powerful in this case. The Henry test (Droesbeker, 1997) also reveals that the $F_{contact}$ distribution is not a Gaussian one (Figure 15) because the obtained points (circle) do not follow a straight line.
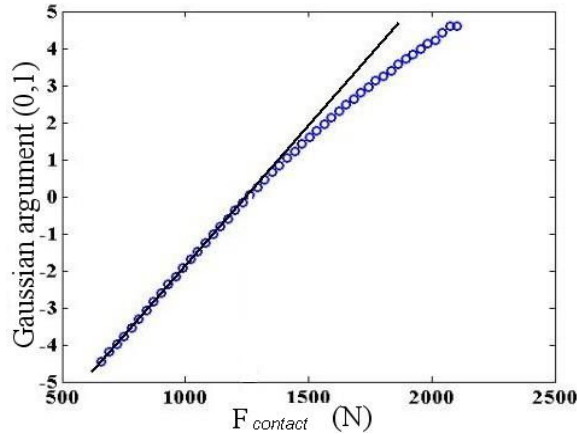

Fig. 15. Henry test for a non-Gaussian distribution.

Indeed, this curve obtained by Henry test deviates from the straight line beyond $F_{contact} \sim 1500\ N$, that means that $F_{contact}$ distribution deviates from a Gaussian distribution from this value.

Percentage of error obtained with Monte Carlo method, coming from Eq. (8), is mentioned for different values of $F_{ultimate}$, see Figure 8. It indicates that Monte Carlo results are probably reliable until $F_{ultimate} \sim 1900\ N$; approximatively 10% error is reached at this level. Beyond this value, % error rapidly increases. The number of simulations becomes insufficient.

On the other hand, results issued from *MC2* and *FORM2-Anal* (only linear terms are kept for $F_{contact}$) are compared on Fig. 16.
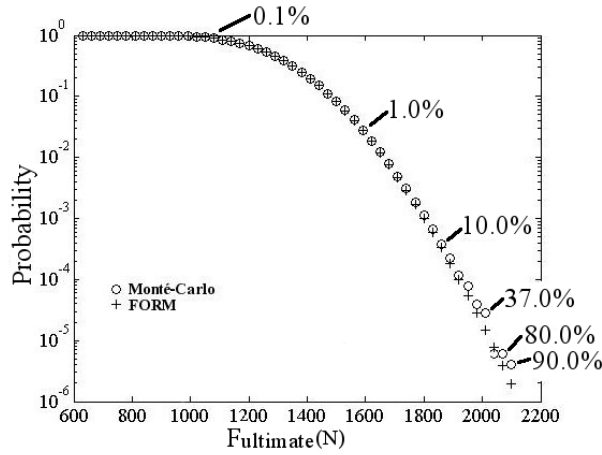
Fig. 16. Probability evolution evaluated by *MC2* and *FORM2-Anal.*

Figure 17 shows the result of the Henry test applied on the distribution representing the polynomial with only the linear terms. The distribution can be now considered as a Gaussian one.
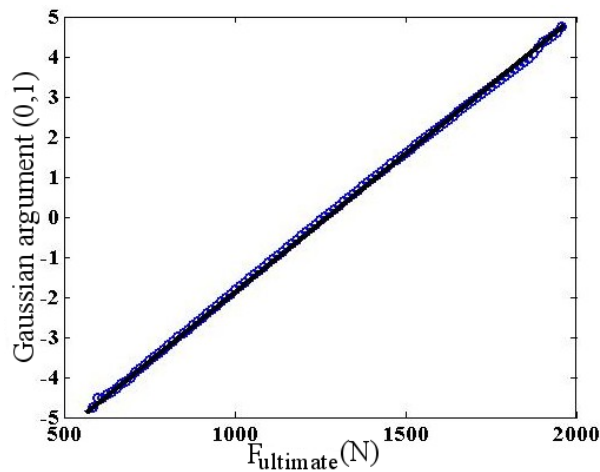

Fig. 17. Henry test for a Gaussian distribution.

Better agreement is observed compared to the previous case (Figure 13). FORM approximated method gives an 'exact' estimation of the failure probability due to the Gaussian nature of all the distributions used in this simulation. Concerning Monte Carlo results, same remarks than previous ones for *MC1* can be done. Results are reliable as much as the number of simulations is in agreement with the expected probability level. The limit of result validity is around $F_{ultimate} \sim 1850\ N$. Beyond this

value, error is over 10% and rapidly increases. On the opposite, *FORM2-Anal* gives correct failure probability estimation without limitation. Indeed, as *FORM2-Anal* results are validated by *MC2* ones for high probability levels (till around P_f=3.10^{-4} corresponding to 10% error), the validation can be extended for small probabilities level as regards FORM properties in this case of linear limit state function.

Moreover, *MC1, FORM1-Anal* and *FORM1-GA* results are finally presented on Figure 18. Those obtained by *FORM1-GA* are encouraging but need to be improved as regards discrepancies for low probability levels. Obviously, this technique should be better in case of complex shapes for the limit state function.

Fig. 18. Comparison between *FORM1-Anal*, FORM1-*GA* and *MC1*.

Finally, this kind of results is very usefull for designers to define some dimensions of the structure for a given probability of failure. For example, in our case the x_3 variable (span) could be considered as a distance between two panel stiffeners. It means that using the methodology developed in this work, it is possible to link some structures data to a level probability in order to estimate a risk.

6. Simulations and results

In this work, a reliability approach for designing composite plates as a part of railway structure under low velocity impact loading is developed. A methodology is presented for this approach. First of all, it consists in using experimental design to approximate correctly the interaction force between the structure and the striker $F_{contact}$ during the impact. Doing so, the limit state function G comparing $F_{contact}$ to a critical one $F_{ultimate}$ can be easily evaluated. The failure probability is then

calculated by FORM determining the reliability factor β either analytically or numerically. Genetic algorithms are principally investigated for the numerical way. Results issued from each technique are compared to Monte Carlo simulations standing for reference. Good agreement are observed. Feasibility of β determination using genetic algorithms is very satisfying and encouraging. In this case of explicit problem formulation, previous algorithms happen to be interesting. Nevertheless, they may be used carefully and the crossing of several methods is indispensable.

Future works are envisaged to consolidate the present approach and to deal with performance functions involving local damage mechanisms in composite such as debonding, delamination, crackings .

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Annexe A. Symbols list

β : Reliability index

P_f : Probability of failure

$F_{contact}$: Contact force between impactor and target

$F_{ultimate}$: Ultimate force

G : Performance function

$\tilde{P}_f$: Estimated failure probability

ϕ : Cumulative Distribution Function (CDF).

GA : Genetic Algorithm

DA : Deterministic Algorithm

GARelDe : Genetic Algorithm for Reliable Design

FORM : First Order Reliability Method

SORM : Second Order Reliability Method

MC : Monte Carlo

Annexe B. Figures list

Annexe C. Tables list