A Novel Approach for Simplification of Industrial Robot Dynamic Model Using Interval Method*

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Abstract— This paper proposes a new approach to simplify the dynamic model of industrial robot by means of interval method. Due to strong nonlinearities, some components of robot dynamic model such as the inertia matrix and the vector of centrifugal, Coriolis and gravitational torques, are very complicated for real-time control of industrial robots. Thus, a simplification algorithm is presented in this study in order to reduce the computation time and memory occupation. More importantly, this simplification is suitable for arbitrary trajectories in whole robot workspace. Furthermore, the method devotes to finding negligible inertia parameters, which is useful for robot model identification. A simulation has been carried out on a test trajectory using a 6-DOF industrial robot model, and the results have shown good performance and effectiveness of this method.

I. INTRODUCTION

Robot manipulators have been increasingly used in various industrial applications in recent years, such as assembly, spray painting, materials machining and welding tasks, improving the productivity, flexibility, and quality. However, for most industrial robots applied to machining and Friction Stir Welding (FSW) process, a high precision and real-time performance can not be achieved. In order to ensure a better tracking performance, many studies about industrial robot control have been performed. Although the machining accuracy can be raised by means of observer-based control or other methods [1], it can not avoid to bring a more complicated control structure. As a result, a great deal of calculation and memory occupation will be carried out in robot controller, affecting the real-time capability. Therefore, low complexity models are essential and model reduction methods are very useful tools.

Industrial manipulators are highly nonlinear, highly coupled and time-varying systems. There exist many model reduction methods available for nonlinear systems [2], including heuristic methods, linearization around equilibrium point or trajectory, balancing using energy functions, balancing empirical Gramians [3], proper orthogonal decomposition, trajectory piecewise-linear approach [4], model reduction through system identification. A computationally efficient reduction method relating to balanced truncation was also proposed in [5]. All these methods can reduce the number of states and improve calculation speed, while the limitation is that they are only applicable to specific trajectories.

Interval method has been widely used in various applications [6], [7], such as parameter and state estimation, robust control and robotics, etc. Using interval analysis, Kieffer and Jaulin [8] proposed a guaranteed recursive nonlinear state estimator, which can solve many actual tracking problems. Its applications to the robust autonomous localization and tracking of mobile robots were presented in [9], [10]. The main limitation of such technique lies in the explosion of complexity with the number of state variables. A similar robust navigation method was also applied to sailboat robots [11], and an interval-based method for the validation of reliable and robust navigation rules was given meanwhile. The authors of [12] combined the interval computation and constraint propagation to tackle some difficult problems in nonlinear identification and robust control.

In the field of industrial robots, a new approach based on interval analysis was developed to find the global minimum-jerk (MJ) trajectory of a robot manipulator within a joint space scheme using cubic splines [13]. The geometric design issue of serial-link robot manipulators with three revolute (R) joints was solved for the first time using an interval analysis method [14]. Gouttefarde et al. [15] presented an interval-analysis-based approach to the wrench-feasible workspace determination of n-DOF parallel robots driven by n or more cables. In [16], a novel approach based on interval method was used to deal with problems of dynamic self-collision detection and prevention for 2-DOF robot manipulator. The forward kinematic map of serial manipulator was extended to intervals using the product of exponential formulation in order to analyze the kinematic errors [17].

As for model simplification using interval method, Martini proposed a method to simplify a 7-DOF helicopter model [18]. During this simplification, the terms related to the components of the inertia matrix and the vector of centrifugal, Coriolis and gravitational torques were first studied from the point of view of specific trajectories, including the normal and helical trajectories. Then all the terms were analyzed by using interval method, which could be generalized to arbitrary trajectories. A simulation for the two trajectories was carried out to demonstrate effectiveness of this method as well. Nevertheless, his method is limited to some specific flight trajectories.

The objective of this paper is to propose a simplification approach for the dynamic model of heavy industrial ma-

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nipulator using interval method. In the model-based control design, simple models are highly preferred. By applying the resulting simplified model to practical control system, the computation time and memory occupation will be greatly reduced, particularly in the observer-based control presented by Qin et al. [1], [19]–[21]. What is more important is that this simplification is suitable for arbitrary trajectories in whole robot workspace. Moreover, the simplification devotes to finding negligible inertia parameters, which is very useful for robot model identification.

This paper is organized as follows: The modeling of an industrial manipulator KUKA KR500-2MT is firstly presented in Section II. In Section III the definition and operation of interval is introduced, followed by the simplification algorithm of robot model and results for whole workspace. In Section IV, a simulation is carried out on a test trajectory to verify the performance of simplification. Section V provides some usage of the simplified model. Finally, Section VI concludes this paper by discussing the advantage of this simplification.

II. ROBOT DYNAMIC MODEL

The industrial robot considered in this study is a serial manipulator KUKA KR500-2MT, as shown in Fig. 1. It has six degrees of freedom, and is composed of six moving links and six revolute joints. We would like to use this robot model to present our simplification method. In order to simplify the modeling of robot, we assume that the tool is directly fixed on link 6 and the spindle of tool is coincident with the axis of joint 6.
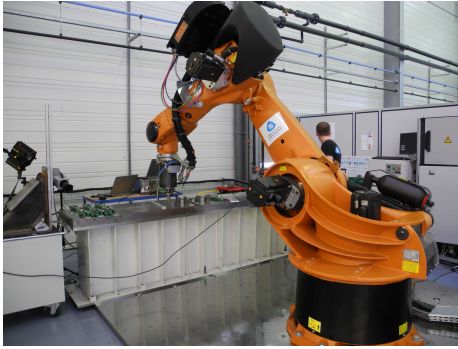

Fig. 1. Robot KUKA KR500-2MT (by courtesy of Institut de Soudure)

On the basis of the research work of Khalil and Dombre [22], the modified Denavit-Hartenberg geometric description is commonly adopted in the modeling of robots. Fig. 2 shows a geometric description of the robot manipulator. After defining the reference frames for the robot, the Modified Denavit-Hartenberg notation can be applied to obtain the geometric parameters, which are listed in Table I. The numerical values of these parameters used for the simplification are given in Table VI of Appendix.

In accordance with this description, the robot motion can be completely described by the vector q of six generalized coordinates: $q = [q_1, q_2, q_3, q_4, q_5, q_6]^T$. The description facilitates to calculate symbolic expressions of the geometric,
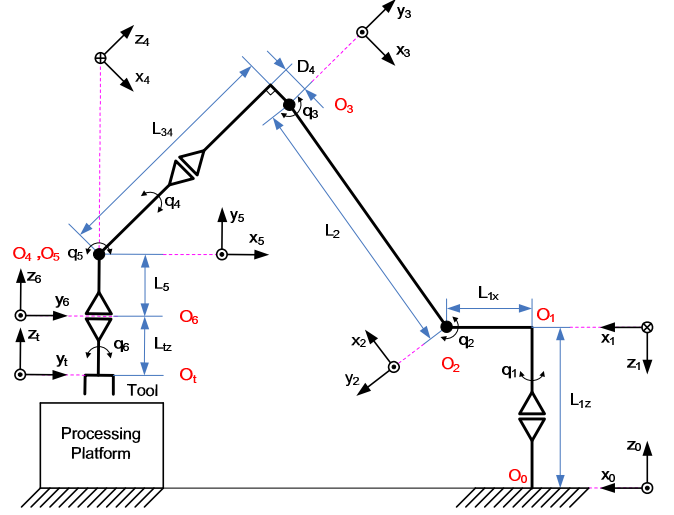

Fig. 2. Geometric description of the robot [1]

TABLE I
GEOMETRIC PARAMETERS OF ROBOT MODEL [1]

j	μ_j	σ_j	α_j	d_j	θ_j	r_j
1	1	0	π	0	q_1	$-L_{1z}$
2	1	0	$\pi/2$	L_{1x}	q_2	0
3	1	0	0	L_2	$\pi/2 + q_3$	0
4	1	0	$-\pi/2$	$-D_4$	q_4	$-L_{34}$
5	1	0	$\pi/2$	0	q_5	0
6	1	0	$-\pi/2$	0	$-\pi/2 + q_6$	$-L_5$
t	0	2	0	0	0	$-L_{tz}$

kinematic and dynamic model of robot with the help of the software *SYMORO+* [23], [24], or other robotic techniques.

Using the Newton-Euler method or the Lagrange equations, one can get the dynamic model of the robot as the following form [1]:

$$M(q)\ddot{q} + H(q,\dot{q}) + F_{fr}(\dot{q}) + J^T(q)F = \Gamma \qquad (1)$$

where $M(q)_{(6\times6)}$ is the symmetric, uniformly positive definite and bounded inertia matrix, $H(q,\dot{q})_{(6\times1)}$ represents the vector of centrifugal, Coriolis and gravitational torques, $F_{fr}(\dot{q})_{(6\times1)}$ is the vector of friction at the robot axis, $F_{(6\times1)}$ is the vector of efforts applied by robot on external environment, $J^T(q)_{(6\times6)}$ is the transposed Jacobian matrix of the tool frame, $\Gamma_{(6\times1)}$ is the vector of gearbox torque, and q, $\dot{q}$ and $\ddot{q}$ represent the robot angular position, velocity and acceleration vectors respectively.

III. SIMPLIFICATION USING INTERVAL METHOD

A. Basic Definitions and Operations of Intervals

The interval denoted by $[a, b]$ is the closed set of real numbers given by:

$$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\} \qquad (2)$$

If the left and right endpoints of an interval X are denoted by $\underline{X}$ and $\overline{X}$ respectively, the *width*, *midpoint* and *absolute value* of this interval X can be defined as follows:

$$w(X) = \overline{X} - \underline{X} \qquad (3)$$

$$m(X) = (\underline{X} + \overline{X})/2 \tag{4}$$

$$|X| = max\{|\underline{X}|, |\overline{X}|\} \tag{5}$$

Let X, Y be real compact intervals and the general form of the interval arithmetic operations are defined as:

$$X \odot Y = \{x \odot y : x \in X, y \in Y\} \tag{6}$$

where $\odot$ stands for one of the basic operations including addition, subtraction, multiplication and division, and here we assume $0 \notin Y$ in case of division. In addition, these operations can be expressed in terms of the endpoints of intervals, and the following rules hold:

$$X + Y = [\underline{X} + \underline{Y}, \overline{X} + \overline{Y}] \tag{7}$$

$$X - Y = [\underline{X} - \overline{Y}, \overline{Y} - \underline{X}] \tag{8}$$

$$X \cdot Y = [min\{\underline{XY}, \underline{X}\overline{Y}, \overline{X}\underline{Y}, \overline{XY}\}, max\{\underline{XY}, \underline{X}\overline{Y}, \overline{X}\underline{Y}, \overline{XY}\}] \tag{9}$$

$$X/Y = X \cdot \frac{1}{Y}, \text{ where } \frac{1}{Y} = [1/\overline{Y}, 1/\underline{Y}] \text{ if } 0 \notin Y \tag{10}$$

Similarly the notion of intervals and interval arithmetic can be extended to include interval vectors and interval matrices as well. More details about intervals are presented in the books of Moore [6] and Jaulin [7].

B. Simplification Algorithm

From (1), we can observe that the expressions of some components are very complicated, such as the inertia matrix $M(q)$ and the vector of centrifugal, Coriolis and gravitational torques $H(q, \dot{q})$. All expressions of these components show strong nonlinearities and a high coupling between the control inputs, which make it difficult to design the control law of robot. Therefore, it is necessary to simplify these expressions in order to acquire a simple model containing essential terms and characteristics of the robot manipulator.

Various model reduction methods for nonlinear systems are existent, however, most of them are only applicable to specific trajectories. According to the specification of robot KUKA KR500-2MT, the range of angular motion, velocity and acceleration for each axis are given in Table II. As shown in this table, all the variables about trajectories in operational workspace are intervals. As a result, interval method will be an easier and more effective way to simplify robot dynamic model compared with other methods. More importantly, the interval-method-based simplification can be definitely applied to arbitrary trajectories in robot workspace.

TABLE II

RANGE OF ROBOT MOTION, VELOCITY AND ACCELERATION

Axis	Angular Motion	Velocity	Acceleration
1	$\pm185°$	$\pm42°/s$	$\pm59°/s^2$
2	$-130°$ to $+20°$	$\pm42°/s$	$\pm43°/s^2$
3	$-94°$ to $+150°$	$\pm42°/s$	$\pm78°/s^2$
4	$\pm350°$	$\pm76°/s$	$\pm42°/s^2$
5	$\pm118°$	$\pm74°/s$	$\pm92°/s^2$
6	$\pm350°$	$\pm123°/s$	$\pm68°/s^2$

For every component of inertia matrix $M(q)_{(6\times6)}$ and vector $H(q,\dot{q})_{(6\times1)}$ in (1), its expression can be described as follows:

$$\begin{aligned} M_{ij} &= f_1(\sin(q_m), \cos(q_n)) \\ H_i &= f_2(\sin(q_m), \cos(q_n), \dot{q}_l) \end{aligned} \tag{11}$$

where $i, j, m, n, l \in \{1, 2, 3, 4, 5, 6\}$, f_1 and f_2 are maps. Thus, a simple component can be taken as an instance to address the simplification algorithm, such as M_{44}, formulated as:

$$\begin{aligned} M_{44} &= ZZ_{4R} + ZZ_6 c_5^2 + 2XY_5 c_5 s_5 + 2L_5 MY_6 c_5 c_6 s_5 \\ &+ 2YZ_6 c_5 c_6 s_5 + XX_{5R} s_5^2 + 2L_5 MX_6 c_5 s_5 s_6 \\ &+ 2XZ_6 c_5 s_5 s_6 + 2XY_6 c_6 s_5^2 s_6 + XX_{6R} s_5^2 s_6^2 \end{aligned} \tag{12}$$

where s_i, c_i denote $\sin(q_i)$ and $\cos(q_i)$ respectively, ZZ_{4R}, ZZ_6, XY_5, MY_6, YZ_6, XX_{5R}, MX_6, XZ_6, XY_6, and XX_{6R} are 10 of 36 regrouped inertia parameters used in robot modeling, and more details can be found in [20], [25]. The whole algorithm process can be divided into 3 steps.

1) Compress the interval

First of all, some trigonometric transformations can be used to rewrite products and powers of sine and cosine functions in the expression, in terms of trigonometric functions with combined arguments. For example:

$$\begin{aligned} 2\cos^2(x) &= 1 + \cos(2x) \\ 2\sin(x)\cos(y) &= \sin(x+y) + \sin(x-y) \end{aligned} \tag{13}$$

Then we carry out the following transformation:

$$a\cos(x) + b\sin(x) = \sqrt{a^2 + b^2} \cos(x - \arctan(a, b)) \tag{14}$$

Accordingly the component M_{44} is transformed as:

$$\begin{aligned} M'_{44} &= \sum_{i=1}^{10} T_i = ZZ_{4R} + \frac{1}{2}ZZ_6 + \frac{1}{2}XX_{5R} + \frac{1}{4}XX_{6R} \\ &+ \frac{1}{4}t_0 \cos(2q_6 - \arctan(-XX_{6R}, 2XY_6)) \\ &+ \frac{1}{8}t_0 \cos(2q_5 + 2q_6 - \arctan(XX_{6R}, -2XY_6)) \\ &+ \frac{1}{8}t_0 \cos(2q_5 - 2q_6 - \arctan(XX_{6R}, 2XY_6)) \\ &+ \frac{1}{2}\sqrt{t_1^2 + t_2^2} \cos(2q_5 + q_6 - \arctan(-t_1, t_2)) \\ &+ \frac{1}{2}\sqrt{t_1^2 + t_2^2} \cos(2q_5 - q_6 - \arctan(t_1, t_2)) \\ &+ \frac{1}{4}\sqrt{t_3^2 + t_4^2} \cos(2q_5 - \arctan(t_3, t_4)) \end{aligned} \tag{15}$$

where $t_0 = \sqrt{XX_{6R}^2 + 4XY_6^2}$, $t_1 = L_5 MX_6 + XZ_6$, $t_2 = L_5 MY_6 + YZ_6$, $t_3 = 2ZZ_6 - 2XX_{5R} - XX_{6R}$, $t_4 = 4XY_5$. As a result, the number of terms in the component is reduced from 16 to 10, and the interval is compressed from $[-8.8580, 306.27]$ to $[76.007, 195.34]$ without any approximation.

2) Remove unimportant terms

The norm of an interval, also called absolute value, can be gained from (5). Since most of terms in the expression are intervals, it is possible to compare them by calculating their norms. Define k_t as a proportional factor, then any term T_i can be neglected if it satisfies:

$$\frac{|T_i|}{g(|T_1|, |T_2|, \cdots, |T_n|)} \leq k_t \qquad (16)$$

where n is total number of terms of a component, g is one of maps which can find the maximum value, sum and root mean square (RMS) value of inputs. In this paper, the maximum value map is adopted. This step can be illustrated by M'_{44} in (15), and the norm ratio of each term to maximum one is provided in Table III, except the first 4 constant terms.

TABLE III
NORM RATIO OF EACH TERM TO MAXIMUM

Term	T_5	T_6	T_7	T_8	T_9	T_{10}
Ratio(%)	33.6	16.8	16.8	2.30	2.30	100

If k_t is chosen as 5%, correspondingly the term T_8 and T_9 in expression (15) will be removed. The simplified component is in the range of $[77.611, 193.74]$, approximately equal to the original one.

3) Neglect unimportant inertia parameters

As a matter of fact, not every inertia parameter has a great influence on norm of a term. Take the term T_{10} in (15) for an instance, the inertia parameter XX_{5R}, XY_5, XX_{6R}, and ZZ_6 are found in the expression. Define k_p as a new proportional factor, then any inertia parameter P can be removed if it meets the following condition:

$$e_p = \frac{|\, |T|_{P=0} - |T|\, |}{|T|} \leq k_p \qquad (17)$$

The result is given in Table IV. Assuming that k_p is assigned to 15%, the inertia parameter XY_5 can be neglected.

TABLE IV
NORM OF TERM AND ERROR AS P=0

P	none	XX_{5R}	XY_5	XX_{6R}	ZZ_6
Norm	34.727	27.726	30.225	26.390	25.211
$e_p(\%)$	0	20.16	12.96	24.01	27.40

C. Programming and Results

In the light of above algorithm, programs are developed by using Mathematica well-known for symbolic and interval calculations. The flow chart of the program can be seen in Fig. 3. The results are valid for the whole workspace. Choosing $k_t = 5\%$ and $k_p = 5\%$, the change in the number of terms and negligible inertia parameters for $M(q)$ and $H(q, \dot{q})$ are respectively listed below:

$$\begin{pmatrix} 238 \\ 112\ 73 \\ 93\ 72\ 47 \\ 89\ 41\ 23\ 10 \\ 56\ 35\ 17\ 5\ 3 \\ 40\ 26\ 10\ 3\ 1\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 8 \\ 3\ 18 \\ 53\ 22\ 20 \\ 27\ 33\ 15\ 8 \\ 52\ 24\ 14\ 3\ 3 \\ 36\ 26\ 10\ 1\ 1\ 1 \end{pmatrix} \quad \begin{pmatrix} 341 \\ 357 \\ 337 \\ 321 \\ 318 \\ 302 \end{pmatrix} \Rightarrow \begin{pmatrix} 97 \\ 10 \\ 46 \\ 107 \\ 88 \\ 136 \end{pmatrix}$$

TABLE V
NEGLIGIBLE INERTIA PARAMETERS

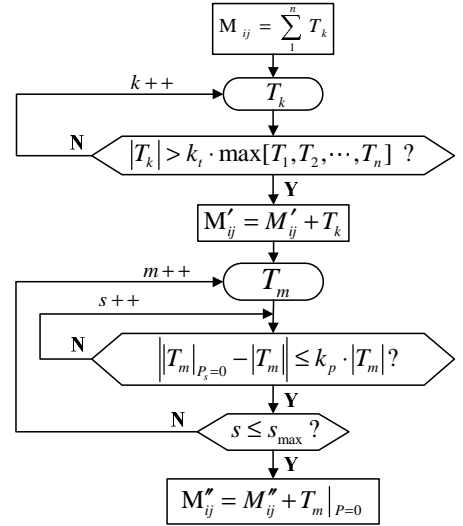| k_t	k_p	$M(q)$	$H(q, \dot{q})$
1%	3%	XY_{3R}, XZ_3	XY_{3R}, XZ_3
1%	5%	$MX_{2R}, XY_{3R}, XZ_3, MX_4$	XY_{3R}, XZ_3, MX_4
5%	3%	XY_{3R}, XZ_3	XY_{3R}, XZ_3
5%	5%	$MX_{2R}, XY_{3R}, XZ_3, MX_4$	XY_{3R}, XZ_3, MX_4

Fig. 3.   Program flow chart

IV. SIMULATION ON A TEST TRAJECTORY

A simulation is carried out on a test trajectory (see Fig. 4), in order to analyze the effectiveness and performance of the simplification method. According to the trajectory data, the interval of angular position, velocity and acceleration can be found in Table VII of Appendix.

Generally, the proportional factors k_t and k_p can be selected intuitively. Calculating the root mean square (RMS) errors can also be considered as a good approach. In this case, three representative components are chosen to find the most appropriate factors, including M_{11}, M_{21}, and M_{22}. Based on the balance between accuracy and simplicity, the number of terms after simplification, RMS error, and value range of component are taken into account.

As a result, we select $k_t = 3\%$, and $k_p = 1\%$. The change in the number of terms for each component of inertia matrix $M(q)$ and the corresponding RMS errors are shown as:

$$\begin{pmatrix} 238 \\ 112\ 73 \\ 93\ 72\ 47 \\ 89\ 41\ 23\ 10 \\ 56\ 35\ 17\ 5\ 3 \\ 40\ 26\ 10\ 3\ 1\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 14 \\ 12\ 26 \\ 61\ 39\ 24 \\ 27\ 37\ 19\ 8 \\ 52\ 32\ 14\ 3\ 3 \\ 40\ 26\ 10\ 3\ 1\ 1 \end{pmatrix}$$
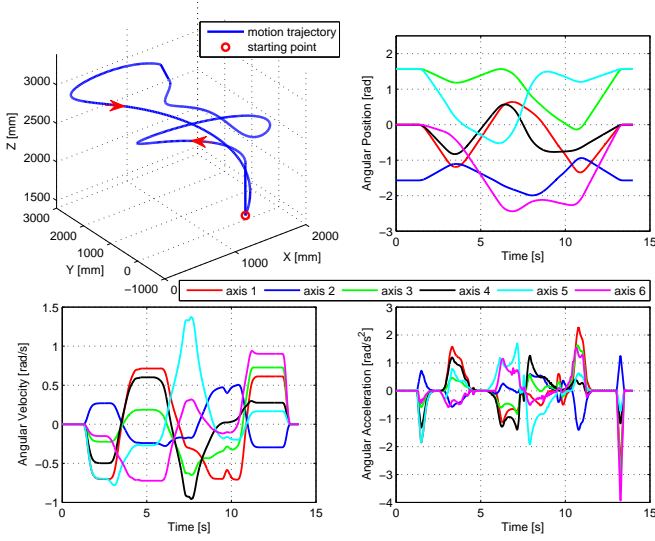
Fig. 4. Angular position and velocity in the trajectory of FSW

Fig. 5. Comparison between simplified and original component in $M(q)$

$$e_{RMS}(\%) = \begin{pmatrix} 1.08 & & & & & \\ 4.46 & 0.77 & & & & \\ 3.07 & 2.48 & 0.83 & & & \\ 9.68 & 1.57 & 0.55 & 0.38 & & \\ 0.91 & 1.60 & 1.56 & 1.29 & 0 & \\ 3.64 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Similarly, by choosing $k_t = 3\%$, and $k_p = 1\%$, the change in the number of terms for each component of vector $H(q,\dot{q})$ and the corresponding RMS errors are given as follows:

$$\begin{pmatrix} 341 \\ 357 \\ 337 \\ 321 \\ 318 \\ 302 \end{pmatrix} \Rightarrow \begin{pmatrix} 147 \\ 46 \\ 66 \\ 153 \\ 116 \\ 170 \end{pmatrix} \quad e_{RMS}(\%) = \begin{pmatrix} 5.89 \\ 4.41 \\ 4.11 \\ 8.38 \\ 4.99 \\ 2.83 \end{pmatrix}$$

Fig. 5 and Fig. 6 show the comparison between simplified and original component for $M(q)$ and $H(q,\dot{q})$. From these figures, it can be seen that on the whole the simplified model is in good agreement with original one, even though some components may not match exactly, such as M_{41}.

However, it should be mentioned that all of the inertia parameters, which are obtained through model identification, have a certain degree of error [20]. Compared with the accuracy of inertia parameter, the error caused by simplification in this simulation is acceptable, and the global RMS errors of $M(q)$ and $H(q,\dot{q})$ are 0.95% and 4.41% respectively.

In fact, the torque $T = M(q)\ddot{q} + H(q,\dot{q})$ in (1) is frequently computed in practical control. Fig. 7 gives the comparison between simplified and original component of torque T, and the RMS errors for each axis are 1.07%, 3.44%, 4.07%, 7.5%, 5.6%, and 2.77% respectively. Furthermore, the global RMS error of torque T is 2.61%. All the above simulation results shows a good performance of simplification.

V. DISCUSSION ON USAGE OF SIMPLIFIED MODEL

Seen from the simulation results, the simplified model is much simpler than the original one while with enough accuracy. As a consequence, the simplified model can be
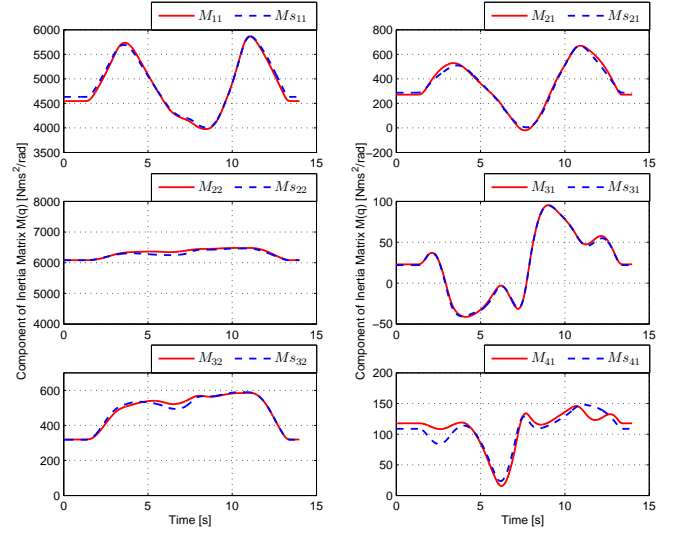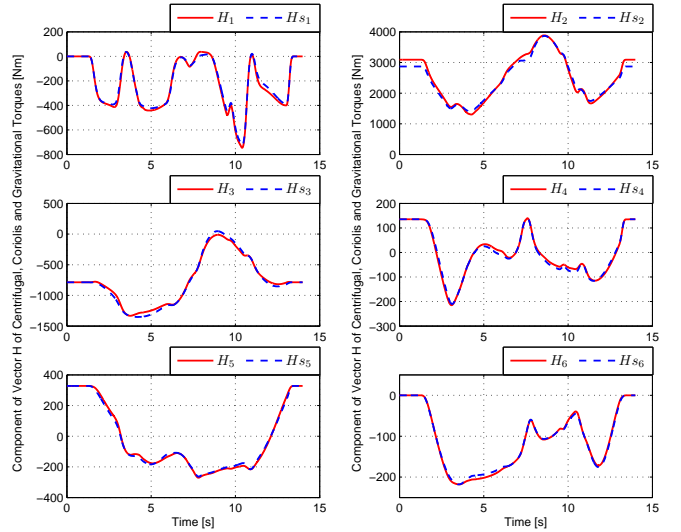

Fig. 6. Comparison between simplified and original component in $H(q,\dot{q})$

applied to many control methods to simplify the control structure and improve the real-time performance, such as the computed torque controller or the observer-based control.

In the research work presented by Qin et al. [1], [19]–[21], as a strong external force is exerted on the robot during the machining or FSW process, the natural stiffness of industrial robot is not sufficient. To compensate the manipulator deformation, a nonlinear observer is designed to estimate the robot states (q, $\dot{q}$, $\ddot{q}$) and the external force F.

In this observer-based control for robot KUKA KR500-2MT, a corrected target position is given to the interpolator of robot controller every 12ms so as to compensate the error between the desired position and so-called real position estimated by a discrete observer, whose sampling time is 1.2ms [21]. The diagram of external deformation compensation of KUKA robot is provided in Fig. 8, which can explain the
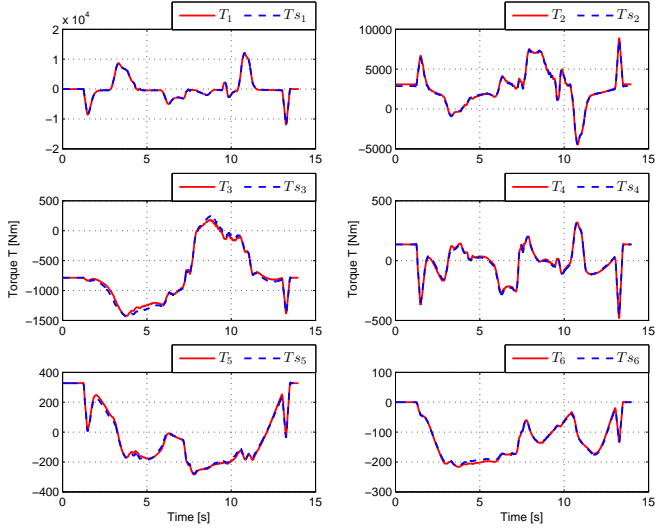
Fig. 7. Comparison between simplified and original torque T

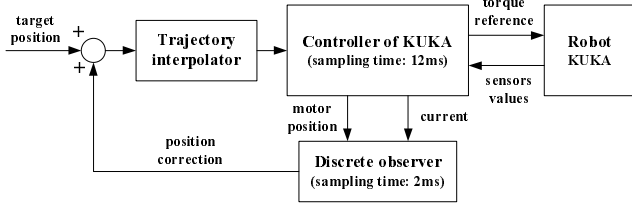reason why the simplification is needed for this control.

Fig. 8. External deformation compensation of KUKA robot [21]

With regard to the estimation of external force in the above observer-based control, it can be obtained by:

$$\widehat{F} = -J^{-T}(q)M(q)\widehat{z_4} \tag{18}$$

where $\widehat{F}$ denotes estimated external force, and $\widehat{z_4}$ denotes observer state 4 [1]. Obviously, the inertia matrix $M(q)$ can be replaced by the simplified one $M_s(q)$ in (18).

In addition, the resulting negligible inertia parameters have a significant meaning for the robot model identification. The identified inertia parameters will be more precise as the negligible inertia parameters are removed [25].

VI. CONCLUSIONS

A new approach using interval method for simplification of industrial robot dynamic model is presented in this paper. From the symbolic dynamic model of a 6-DOF industrial manipulator, all the expressions of components in inertia matrix $M(q)$ and vector $H(q,\dot{q})$ show complexities and strong nonlinearities, besides, a high coupling between control inputs also exists. Thus, a simplification algorithm is proposed to make the robot model much simpler. As a simple component of inertia matrix, M_{44} is taken to illustrate the entire process. The results for the whole workspace are suitable for arbitrary trajectories. A simulation on a test trajectory is carried out and the comparisons between simplified and original components of $M(q)$, $H(q,\dot{q})$ and

T are performed. A good accuracy of the simplified model is shown, demonstrating the effectiveness and good performance of the method. The simplified model can be applied to the observer-based control, and it also devotes to finding negligible inertia parameters, which is very useful for robot model identification.

APPENDIX

TABLE VI
NUMERICAL VALUES OF ROBOT GEOMETRIC PARAMETERS

Length	L_{1x}	L_{1z}	L_2	L_{34}	D_4	L_5	L_{tz}
(m)	0.500	1.045	1.300	1.025	0.055	0.235	0.435

TABLE VII
ACTUAL INTERVAL OF THE ANGULAR POSITION AND VELOCITY

Axis	$AngularPosition(rad)$	$Velocity(rad/s)$	$Velocity(rad/s^2)$
1	[-1.3457,0.6393]	[-0.7091,0.7141]	[-2.5811,2.2603]
2	[-1.9929,-0.9411]	[-0.2951,0.5056]	[-1.4035,1.2476]
3	[-0.1359,1.5707]	[-0.6485,0.7263]	[-3.0617,1.6341]
4	[-0.8298,0.5699]	[-0.9569,0.6014]	[-1.3964,1.2582]
5	[-0.5254,1.5709]	[-0.7805,1.3723]	[-1.9199,1.6932]
6	[-2.4747,0]	[-0.7331,0.9391]	[-3.9371,1.5219]

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