Science Arts & Métiers (SAM) is an open access repository that collects the work of Arts et Métiers ParisTech researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: https://sam.ensam.eu
Handle ID: .http://hdl.handle.net/10985/8897

To cite this version:

Any correspondence concerning this service should be sent to the repository Administrator: archiveouverte@ensam.eu
A method for the field assessment of rolling resistance properties of manual wheelchairs

Joseph Bascoua, Christophe Saureta, Hélène Pilleta, Philippe Vaslinb, Patricia Thoreuxa and François Lavasteac

aLBM, Arts et Métiers ParisTech, 151 Bd de l’Hôpital, Paris 75013, France; b3 Clermont Université, Université Blaise Pascal, LIMOS, BP 10448, Clermont-Ferrand 63000, France; cCERAH/INI, Antenne de Créteil, 47 rue de l’Echat, Créteil 94000, France

(Received 22 December 2010; final version received 12 September 2011)

This article presents an examination and validation of a method to measure the field deceleration of a manual wheelchair (MWC) and to calculate the rolling resistances properties of the front and rear wheels. This method was based on the measurements of the MWC deceleration for various load settings from a 3D accelerometer. A mechanical model of MWC deceleration was developed which allowed computing the rolling resistance factors of front and rear wheels on a tested surface. Four deceleration sets were conducted on two paths on the same ground to test the repeatability. Two other deceleration sets were conducted using different load settings to compute the rolling resistance parameters (RPs). The theoretical decelerations of three load settings were computed and compared with the measured decelerations. The results showed good repeatability (variations of measures represented 6–11% of the nominal values) and no statistical difference between the path results. The rolling RPs were computed and their confidence intervals were assessed. For the last three sets, no significant difference was found between the theoretical and measured decelerations. This method can determine the specific rolling resistance properties of the wheels of a MWC, and be employed to establish a catalogue of the rolling resistance properties of wheels on various surfaces.

Keywords: 3D accelerometer; deceleration test; rolling resistance; rolling resistance parameters; wheel; wheelchair

1. Introduction

During manual wheelchair (MWC) locomotion, the user expends energy to generate joint forces and torques, which are transferred to the MWC. The user’s mobility thus depends on muscle strength, locomotion techniques, MWC properties (inertial parameters, adjustments, etc.) and substantial sources of energy loss by the MWC, i.e. rolling, turning, bearing and aerodynamic resistances (Cooper 1990; Hofstad and Patterson 1994). In the daily life of MWC users, the ability for straight displacement is considerably important. In this condition, the turning, bearing and aerodynamic resistances can be neglected with regard to rolling resistance (Hofstad and Patterson 1994), which depends on wheels’ properties (material, width, radius, etc.), floor type (hardness and roughness) and loads applied on front and rear wheels, for instance. Besides, the rolling resistance was proved to increase when the mass of the loaded MWC is brought forward (de Saint Rémy et al. 2003, de Saint Rémy, 2005; Sauret et al. 2006, 2009, 2010), due to differences in the radii of the front and the rear wheels (Brubaker et al. 1986). Hence, the energy loss by a MWC during propulsion would depend on both the total mass of the MWC-user system and its fore–aft distribution (de Saint Rémy et al. 2003). Thus, characterising the rolling resistance properties of several MWCs should account for these parameters.

Several papers focused on the assessment of MWC rolling resistance using various techniques: several authors measured the global drag force, with a force sensor, sustained by a MWC (loaded with a MWC user or a dummy) rolling on a motor-driven treadmill (Kauzlarich and Thacker 1985; Brubaker et al. 1986; van der Woude et al. 1986; de Groot et al. 2006); others determined the rear wheel deceleration on a roller ergometer (Theisen et al. 1996; Faupin et al. 2004; Kwarciak et al. 2009); or determined the rolling coefficients of front and rear wheels from measurements of a force plate during a deceleration test performed with a MWC loaded with a MWC user (Lemaire et al. 1991). Unfortunately, even if these techniques allowed testing different types of wheels, they did not allow testing different floors. Thus, the results remained confined to the materials of the treadmill belt, the rollers or the force-plate covering. Other techniques, based on deceleration tests (or coast down test) performed in the field, were also described. Coutts (1992, 1994) computed the deceleration of a MWC loaded with a user from a second-order time differentiation of the rear wheels’ angular positions (four measurements per turn); others computed the MWC deceleration using the movement differential equations from the time measurement to cross a known distance (Hoffman et al. 2003); or by directly measuring the deceleration from 3D accelerometer with a

*Corresponding author. Email: joseph.bascou-8@etudiants.ensam.eu
MWC loaded by artificial masses (Vaslin and Dabonneville 2000; de Saint Rémy et al. 2003). All these field techniques allowed testing various MWCs equipped with different wheels and on different floors. However, computing deceleration from rear wheels angular displacements (Coutts 1992, 1994) required the use of digital filters before differentiating the data that could alter the deceleration value. In the technique developed by Hoffman et al. (2003), the limit was the assessments of both the initial instantaneous velocity and the actual distances travelled by the MWC achievable with their equipment (photo-electric cells), which did not provide sufficient accuracy. Hence, the technique developed by Vaslin and Dabonneville (2000) would provide better results than the others listed above in quantifying the rolling resistance of various types of wheels and floors.

From another perspective, most methodologies used in the past did not account the influences of both the mass and its fore–aft distribution (Kauzlarich and Thacker 1985; Brubaker et al. 1986; van der Woude et al. 1986; Coutts 1992, 1994; Hoffman et al. 2003). Indeed, few authors distinguished the loads on front and rear wheels (Lemaire et al. 1991; Sauret et al. 2006, 2009). These authors have thus characterised the rolling resistance properties of a MWC by two rolling coefficients (front–rear wheels), which are specific to each wheel–floor couple. In this manner, it was possible to assess the rolling resistance for various masses and fore–aft distributions of this mass.

To characterise the rolling resistance properties of various MWCs on different floors, a good solution could be the measurement of MWC deceleration with a 3D accelerometer during field deceleration tests, then the computing of front and rear wheels’ rolling resistance properties, based on the previous works of Vaslin and Dabonneville (2000), de Saint Rémy et al. (2003) and Sauret et al. (2006, 2009). However, before applying this technique in an extensive way to compare several MWCs or floors, it is important to evaluate the accuracy of the provided results. This study completed the description of the method, provided validation of the repeatability of the tests and assessed the range of potential errors.

2. Materials and methods

2.1 Modelling of rolling resistance

The MWC, loaded with a fixed mass and decelerating on a straightforward motion under the only action of the rolling resistance – neglecting bearing, slipping and air resistances (Hofstad and Patterson 1994; Van der Woude et al. 2006) – was modelled as presented in Figure 1. The mechanical model that links the deceleration of the global center of mass (COM) \( \gamma_g \) to both forces and torques exerted on the system (MWC + artificial masses) is detailed in Appendix A and is written as follows (see symbols description in Table 1):

\[
\gamma_g = -mg \left( \frac{h}{r_f w_b} + \frac{h}{r_r w_b} + \frac{h}{r_f r_r w_b} \right) \left( 1 + \frac{h}{w_b} \right) + \left( m + \frac{h}{r_f w_b} + \frac{h}{r_r w_b} \right) \left( \frac{h}{r_f} - \frac{h}{r_r} \right) \frac{h}{w_b}.
\]

This equation is an exhaustive model of rolling resistance. However, it can be correctly approximated (<3% error) by the following expression, leaving out the negligible terms (see details in Appendix B):

\[
\gamma_g = -g \left( \frac{\lambda_f d_r}{r_f w_b} + \frac{\lambda_r d_l}{r_r w_b} \right).
\]

In this expression, \( \lambda_f \) and \( \lambda_r \) are the rolling resistance parameters (RPs) of the front and rear wheels, respectively. They represent the fore–aft distance between the theoretical centre of rotation of the wheel on the floor.
(normal projection of wheel centre on the floor) and the centre of pressure in the contact area where the resulting ground reaction force is applied. As a consequence, the ground reaction force creates a resisting moment with respect to the theoretical centre of rotation on the floor, namely the moment of rolling resistance. The distances $\lambda_f$ and $\lambda_r$ are a consequence of the material inelastic properties of both wheels and floor (i.e. hysteresis phenomenon) and characterise the contact between the wheel and the ground.

The ratio between the rolling RP ($\lambda$) and the wheel radius ($r$) is called the rolling resistance factor and represents the effective rolling resistance property of a wheel. Then, the rolling resistance factor characterises the wheel.

Finally, the resultant force of rolling resistance ($F_{\text{roll}}$), characterising the MWC, can be obtained by multiplying Equation (2) by the total mass ($m$) to give a formulation that is consistent with those already expressed (Cooper 1990; Sauret et al. 2009):

$$F_{\text{roll}} = mg = -\left(\frac{\lambda_f}{r_f} W_f + \frac{\lambda_r}{r_r} W_r\right).$$

Furthermore, Equation (2) can also be written using the mass proportion on the front and rear wheels:

$$\gamma_G = -g \left(\frac{m_f}{r_f} p_f + \frac{m_r}{r_r} p_r\right).$$

### 2.2 Experimental protocol

To reproduce the hypothesis leading to Equation (4) for a given MWC and floor, the selected MWC was loaded concatenating additional masses on the seat and close to the floor. This way, the MWC oscillations in horizontal and sagittal plane, due to the frame deformation and the pushing of the MWC, were limited. Its deceleration during free-wheeling phase was then measured.

The estimation of the rolling resistance factors in Equation (4) required the measurement of the other values. The resulting loads on front wheels and on rear wheels were measured with a specific large weight-scale platform (resolution: 0.05 kg). A gravitational acceleration value of 9.81 m/s$^2$ was used. The wheel radii were measured with a calliper rule. The deceleration value during the free-wheeling phase associated with this load repartition was obtained by conducting various deceleration tests (see below) and data processing.

#### 2.2.1 Deceleration tests

The deceleration test provided a deceleration value for the free-wheeling phase, and consisted in pushing the MWC and allowing it to decelerate along a straight corridor, measuring the deceleration during this time.

During the acquisition, various phases have to be observed (1) static phase: lasts for 2 s and is used for data processing; (2) push phase: the MWC is manually pushed to 1–3 km/h; (3) free deceleration phase; (4) stop phase: as the deceleration length was limited, the MWC was manually stopped after a 4-m long deceleration phase and (5) static phase: used for data processing.

The deceleration value was measured during the free deceleration phase (phase 3) using a wireless 3D accelerometer (Beanscape AX-3D, Beanair, Neuville-sur-Oise, France, sensitivity: $\pm 2$ g) fixed on the additional masses (a thin foam was used to limit the sensor vibrations) and at a 100 Hz frequency (Vaslin and Dabonneville 2000). Caution was taken to align the accelerometer x-axis with the travel direction (see x-axis of the reference frame in Figure 2).

The start and stop positions and the trajectory as well as the angular start position of the rear wheels (valves down) were controlled; the test was conducted in a narrow lane of 60-cm width drawn on the floor and was rejected if the MWC deviated from this lane (Example: lane A in Figure 3).

#### 2.2.2 There-and-back deceleration

To overcome the limit due to the unevenness of the ground, a there-and-back procedure was adopted: for each deceleration test in one way, another deceleration test was conducted on the way back (Coutts 1991, 1994; Sauret et al. 2010), keeping the same deceleration path. Hence,
the tests were always paired, with half of the tests made in one direction and the other half on the reverse direction, providing one deceleration value for each pair.

2.2.3 Sets deceleration for each load repartition
A set of there-and-back decelerations allowed defining the deceleration value associated with Equation (4) to a given load distribution. Within a set, the load repartition remained constant (the position of the additional mass did not vary) and the MWC deceleration was evaluated by conducting various there-and-back tests on a horizontal floor. In our study, 10 there-and-back procedures were performed for each set, providing 10 deceleration values. One set was done for every load distribution presented in Table 2.

2.3 Data processing: deceleration and rolling resistance properties computation
2.3.1 Deceleration test processing
When the $x$-axis of the 3D accelerometer was perfectly aligned with the travel direction, a 1D accelerometer was sufficient. Unfortunately, it is impossible to perfectly align manually the $x$-axis with the travel direction both in the sagittal and in the horizontal planes (Figure 2). As the errors due to small misalignments in the horizontal plane could be neglected, those caused by misalignments in the sagittal plane (Figure 2) could induce large errors on MWC deceleration measure, due to the action of the gravitational acceleration.

Therefore, to correct misalignments that occurred during the deceleration test, various steps of signal processing were applied to the raw data:
First step. A rotation matrix was defined to transform the accelerometer frame ($R_{\text{acc}}$) to the measurement frame ($R_{\text{measure}}$) and was defined as follows: the gravity measurement during the first static phase (1st part) defined the vertical $y_{\text{measure}}$-axis (Vaslin and Dabonneville 2000; de Saint Rémy et al. 2003); the transversal $z_{\text{acc}}$-axis was the same as the transversal $z_{\text{measure}}$-axis; the $x_{\text{measure}}$-axis, pointing in the travel direction, was defined by the cross product of $y_{\text{measure}}$ by $z_{\text{measure}}$. This axis was perfectly horizontal at the beginning of the measure and was the one used to measure the MWC deceleration. The raw acceleration vector was then transformed using the rotation matrix into the measure vector. The components of this vector were the deceleration value along $x_{\text{measure}}$-axis, the gravity acceleration along $y_{\text{measure}}$-axis and the MWC transversal oscillations along $z_{\text{measure}}$-axis (equal to zero in theory).

Second step. The horizontal velocity was calculated by a first-order time integration of the fore–aft deceleration (along $x_{\text{measure}}$-axis) from the start of the push phase (2nd part) to the complete stop of the MWC (beginning of the 5th part). A constant value was subtracted to the $x_{\text{measure}}$-data to obtain a null velocity at the end of the movement (see Figure 4). This correction corrects small misalignments of the $x_{\text{measure}}$-axis with the deceleration vector, which otherwise induce a drift in the measured velocity.

Third step. The deceleration phase was manually identified from maximal velocity (end of the push phase) until the beginning of the stop phase (characterised by a break in the velocity decrease). The mean deceleration value (along $x_{\text{measure}}$-axis) during the deceleration phase was then calculated. This deceleration value was considered as the MWC centre of mass deceleration during the free deceleration phase of the test.

### 2.3.2 There-and-back set processing

To obtain the deceleration value for each there-and-back procedure, the two decelerations obtained for the pair of deceleration tests were averaged, which allowed cancelling out the tiny slope effect that always exists on an apparently flat ground.

Prior to computing the mean deceleration value of each set (composed of 10 there-and-back deceleration values), the outliers identified using the Box and Whiskers Plots method (Le Guen 2001) were rejected.

### 2.3.3 Wheel rolling resistance factors computation

From the simplified Equation (4), knowing the wheel radius, the load distribution and the MWC COM
deceleration for various load conditions, a set of equations could be formulated, in which only the rolling RPs $\lambda_f$ and $\lambda_r$ are the unknown variables, and were assumed to be unchanged when the loads on each wheel varied.

The set of equations to be solved was presented in the following system:

$$\frac{\lambda_f}{r_f} P_{f1} + \frac{\lambda_r}{r_r} P_{r1} = \frac{\gamma_i}{-g},$$  \hspace{1cm} (5)

where the indices $i$ represented the equation set number.

This system of Equations (5) could be expressed in a matrix form:

$$\begin{bmatrix}
P_{f1} & P_f \\
P_{f2} & P_{r1} \\
\vdots & \vdots \\
\end{bmatrix}
\begin{bmatrix}
\frac{\lambda_f}{r_f} \\
\frac{\lambda_r}{r_r} \\
\vdots \\
\end{bmatrix}
= -\frac{1}{g} \begin{bmatrix}
-\gamma_{1f} \\
-\gamma_{1r} \\
\vdots \\
\end{bmatrix}.$$  \hspace{1cm} (6)

Here, the wheel radii were known, but were integrated in the unknown matrix to maintain a simple equation system. The unknown elements were then the rolling resistance factors.

More generally, the system could then be expressed by

$$[M_D] \cdot [M_{RF}] = \frac{-1}{g} \cdot [M]_I,$$  \hspace{1cm} (7)

where $M_D$ is the distribution matrix, $M_{RF}$ is the matrix of unknowns and $M_I$ is the acceleration matrix.

This can be solved by the following equation, provided that the determinant of $([M_D]^T[M_D])$ is not null, then the matrix invertible is

$$[M_{RF}] = \frac{-1}{g} \cdot ([M_D]^T[M_D])^{-1} \cdot [M_D]^T \cdot [M]_I.$$  \hspace{1cm} (8)

In reality, as the measurements suffer from approximations, the $([M_D]^T[M_D])$ matrix has to be well conditioned, which means that the resulting unknown matrix computation need not change significantly when a random perturbation is placed in the distribution or in the acceleration matrix (Cabane 1998). As two unknowns are present in the system, at least two sets of equations are necessary to solve the system, which means two load conditions and two deceleration values. If more load conditions are considered, the system will be overabundant and the results will be averaged: solved through a root mean square regression, the system becomes less sensitive to small errors when the number of points rises.

2.3.4 Wheel rolling RPs

The rolling RPs could be calculated by multiplying the rolling resistance factors of the front and rear wheels by their respective radii. The values of the rolling resistance factors and the rolling RPs were considered independent of the load applied on each wheel.

2.3.5 Confidence interval on decelerations

The normal distribution of the deceleration values within a set was checked with the Shapiro–Wilk test ($p = 0.05$) and the 95% confidence interval was finally calculated using the Student law (Rakotomalala 2008), which allows
the normality of the there-and-back decelerations was assumed. Two more sets were performed on corridor A (S1 and S2 with S1 and S2), significant differences were found between deceleration tests conducted in the reverse directions. These differences ranged from 0.006 (S1B) to 0.012 m s\(^{-2}\) (S1 and S2B), and in the Student t-test from 2.2 to 5.74.

Considering the there-and-back decelerations (gathering outward and forward deceleration tests), the set decelerations ranged from 0.045 to 0.068 m s\(^{-2}\) for S1B and S2B, respectively. The intra-set variability, expressed through the standard deviation, ranged from 0.003 (S1 and S2B) to 0.004 m s\(^{-2}\) (S1B and S2B). Considering the 95% confidence interval, the measurement uncertainty on deceleration ranged from ±0.004 to ±0.005 m s\(^{-2}\), which was 6–11% of the nominal value. Comparing corridor A with corridor B, the mean decelerations differed by 0.003 m s\(^{-2}\) (between S1 and S1B) and 0.001 m s\(^{-2}\) (between S2 and S2B); however, these differences were insignificant (Student t-test = 1.42 and 0.49, respectively).

The results of sets S1 and S1 are presented in Table 2. The computation of rolling resistance factors using sets S1, S2, S3 and S4 decelerations provided the rolling resistance factors and the Monte Carlo simulation provided an estimation of the confidence intervals: \(\lambda_{\text{front}}/l_{\text{front}}\) (±2SD) = 9.8 \times 10^{-3} (±1.1 \times 10^{-3}) and \(\lambda_{\text{rear}}/l_{\text{rear}}\) (±2SD) = 2.6 \times 10^{-3} (±0.8 \times 10^{-3}). The rolling RPs could then be calculated: \(\lambda_{\text{front}}\) (±2SD) = 0.6 \times 10^{-3} m (±0.03 \times 10^{-3}) and \(\lambda_{\text{rear}}\) (±2SD) = 0.8 \times 10^{-3} m (±0.1 \times 10^{-3}).

The rolling resistance factors and the load conditions of sets S5, S6 and S7 were used to predict their decelerations: 0.061 m s\(^{-2}\) (±0.009) for set S5, 0.052 m s\(^{-2}\) (±0.009) for set S6 and 0.062 m s\(^{-2}\) (±0.009) for set S7. The differences between the computed and the measured decelerations for S5, S6 and S7 were 0.007, 0.006 and 0.004 m s\(^{-2}\), respectively, which were lower than the confidence intervals of each set deceleration.

Multiplying the deceleration values obtained for each set by the total masses yielded drag forces ranging between 2.7 N (58 kg, 37% on the front wheels) and 6.9 N (90.8 kg, 69% on the front wheels).

### 3. Discussion

Owing to the proposed method, the MWC decelerations could be obtained for various sets of load conditions and were consistent with previous studies (Coutts 1991; de Saint Rémy 2003; Sauret et al. 2009). The study underlined the significant influence of the load distribution on the deceleration: for example a 52% increase in the deceleration was observed when the load repartition varied
from 29% ($S_1$) to 64% ($S_2$), although the total mass remained the same. These results were expected and consistent with previous results obtained by de Saint Rémy et al. (2003) and Sauret et al. (2009, 2010).

The results also showed a good repeatability in the deceleration tests when performed in the same direction, but showed differences between tests in one way and tests in the way back. This underlines the significant influence of small floor deformations on the measured deceleration and the need for there-and-back procedures, with which the method proved to be sufficiently robust to conduct experiments on different paths on the same ground without altering the results (providing the ground properties are the same, as shown by the comparison of the sets $S_1$, $S_{1B}$, $S_2$ and $S_{2B}$ for different corridors). This novel result allows the comparison of wheel properties of a MWC on various grounds (concrete, carpet, etc.), ensuring that differences in deceleration values are directly correlated with the ground material properties, rather than its deformities.

To the authors’ knowledge, the rolling resistance factors and parameters of the front and rear wheels of MWC were calculated only once before (Sauret et al. 2006, 2009): the rolling resistances found in our study were lower, but this could be explained by differences in the wheel and ground types (concrete ground vs. athletic track ground). The calculation of predicted decelerations for sets $S_5$, $S_6$ and $S_7$ and their comparison with the measured deceleration showed the validity of the proposed model. The rolling resistance factors were significantly higher on the front wheels than on the rear wheels for the tested MWC, which is consistent with the increase in rolling resistance with the front wheels distribution of the total mass. This could be explained by the mechanical model of rolling resistance: the front wheels’ radii were five times smaller than the rear wheels’ radii and their rolling RPs were quite the same ($0.6 \times 10^{-3}$ m vs. $0.8 \times 10^{-3}$ m). Therefore, the front wheel rolling resistance factor, which is the ratio of the rolling RP to the radius of the front wheel, was four times smaller than that of the rear wheel one. Taking into account the confidence intervals, the front and rear rolling resistance factors could be distinguished: the two standard deviations on the rolling RPs were 6% of the nominal value for the front wheels and 15% for the rear wheels, which must be taken into account when comparing the two wheels on the same ground or two grounds for the same wheels.

The rolling drag forces were in accordance with those found by Coutts (1992, 1994) and Brubaker et al. (1986). However, the use of the drag force to compare the wheelchairs must be handled with extreme caution, as it mainly depends on the load distribution (de Saint Rémy et al. 2003).

4. Conclusion

This study completed the work of previous researches on the deceleration method (Coutts 1991; de Saint Rémy et al. 2003; Sauret et al. 2006, 2009), and proved the interest and the reliability of this technique in assessing the effect of the ground and front and rear wheel choice on the MWC deceleration and rolling drag force.

The mechanical model allowed the front and rear wheels rolling resistance factors and parameters to be computed with an acceptable accuracy. The use of this method could allow the creation of a database of the rolling resistance properties of various wheels on different surfaces: the rolling drag force of a wheelchair could then be calculated from the front and rear wheel types, the ground type and the load distribution. This database would allow a comparison between MWC on a defined floor, according to the load distribution and from an energetic point of view.

Acknowledgements

The authors would like to thank the French National Research Agency (ANR) for its financial support to the SACR-FRM project (ANR-06-TecSan-020) and to the CERAH for the loan of all the manual wheelchairs evaluated in this work.

References


Appendix A

This appendix aims to develop the mechanical model providing Equation (1) carrying on the COM deceleration of the loaded MWC during the deceleration phase of a coast down test and is based on Figure 1: the sagittal plane is considered and the front and rear wheels are treated as pairs.

At first, considering the loaded MWC (frame + wheels) during this phase, the exterior forces applied are the total weight (\(W\)) and the ground reaction forces on the front (\(R_f\)) and rear wheels (\(R_r\)); applying the second law of Newton on the system equation along the fore-and-aft and the vertical directions gives

\[
\sum F_{x,\text{ext}} - W = m \ddot{y}_G \iff R_{f_x} + R_{r_x} = m \ddot{y}_G, \quad (A1)
\]

\[
W + R_{f_N} + R_{r_N} = 0, \quad (A2)
\]

where \(R_{f_x}\) and \(R_{r_x}\) are the fore-and-aft components of the ground reaction forces applied on the front and rear wheels, respectively; \(R_{f_N}\) and \(R_{r_N}\) are the normal components, \(m\) is the total mass and \(\ddot{y}_G\) is the fore-and-aft COM deceleration of the loaded MWC.

Considering the front wheels and their centre \(O_f\), the equality of the torque of the exterior forces in \(O_f\) with the angular momentum variation in \(O_f\), projected on \(z\)-axis, gives

\[
\sum M_{z,\text{ext}} - \text{front wheels} = I_z \text{front wheels} \times \dot{\Gamma}_z = I_f \times \dot{\Gamma}_z,
\]

where \(I_f\) is the front wheel inertia along \(z\)-axis and \(\dot{\Gamma}_z = \gamma_G / r_i\) is the wheel angular acceleration.

The torque of exterior forces can be expressed by

\[
\sum M_{z,\text{ext}} - \text{front wheels} = M_{z,\text{frame}} - \text{front wheels} + M_{z,\text{ground}} - \text{front wheels}
\]

\[= 0 + M_{z,\text{ground}} - \text{front wheels} + (O_f A_f \times F_{\text{ground}} - \text{front wheels}) \cdot z
\]

\[= 0 + 0 + (-r_i \cdot y + \lambda \cdot x) \times (R_{f_N} \cdot y + R_{r_N} \cdot x) \cdot z
\]

\[= r_i R_{f_x} + \lambda R_{r_N}.
\]

When the MWC rolls without slipping on the ground, \(R_{f_x}\) and \(R_{r_x}\) can be expressed by the next equation, where the first part concerns the rolling resistance and the second part concerns the angular momentum variation:

\[
R_{f_x} = -\frac{\lambda_f}{r_i} R_{f_N} - \frac{I_f}{r_i^2} \gamma_G, \quad (A3)
\]

\[
R_{r_x} = -\frac{\lambda_r}{r_r} R_{r_N} - \frac{I_r}{r_r^2} \gamma_G, \quad (A4)
\]

where \(\lambda_f\) and \(\lambda_r\) are the front and rear wheels' RP, \(r_i\) and \(r_r\) are the front and rear wheels' radii and \(I_f\) and \(I_r\) are the moment of inertia along \(z\)-dimension of the two front wheels and the two rear wheels, respectively.

Using the last two Equations (A3 and A4) in Equation (A1) then gathering the terms in \(\gamma_G\) allows linking the normal ground reaction forces to the COM acceleration of the loaded MWC:

\[
-\frac{\lambda_f}{r_i} R_{f_N} - \frac{\lambda_r}{r_r} R_{r_N} = \left( m + \frac{I_f}{r_i^2} + \frac{I_r}{r_r^2} \right) \gamma_G. \quad (A5)
\]

Then, replacing \(R_N\) from Equation (A2) in Equation (A5) allows expressing \(R_{IN}\):

\[
R_{IN} = -\frac{\lambda_f}{r_i} \left( \frac{r_i r_f}{\lambda_f r_f - \lambda_r r_i} \right) W + \left( m + \frac{I_f}{r_i^2} + \frac{I_r}{r_r^2} \right) \times \left( \frac{r_i r_f}{\lambda_f r_f - \lambda_r r_i} \right) \gamma_G.
\]

(A6)

In the second time, the sum of the torques acting on the loaded MWC and expressed at the COM is equal to the resulting dynamic momentum, which is drastically simplified with a MWC loaded with additional masses. So, following the transversal direction:

\[
(d_i - d_r)/2 R_{IN} = -((w_b + \lambda_l - \lambda_r) R_{IN} - (d_i - d_r) W)
\]

(A7)

where \(d_i\) and \(d_r\) are the distance between the COM and the front and rear wheels centres, respectively (\(d_i + d_r\) is the wheelbase \(w_b\)), and \(h\) is the height of the COM with respect to the ground.

Then, using Equations (A1) and (A2) in A7 gives

\[
\left( m + \frac{I_f}{r_i^2} + \frac{I_r}{r_r^2} \right) h \gamma_G = -((w_b + \lambda_l - \lambda_r) R_{IN} - (d_i - d_r) W).
\]

(A8)

Using Equation (A6) in Equation (A8) to replace \(R_{IN}\) gives

\[
\left( w_b + \lambda_l - \lambda_r \right) \left( m + \frac{I_f}{r_i^2} + \frac{I_r}{r_r^2} \right) r_i r_f / (\lambda_f r_f - \lambda_r r_i) W.
\]

Then, replacing the previous equation by \((\lambda_2 r_1 - \lambda_1 r_2)\) and dividing by \(r_1 r_2 w_b\) gives

\[
\left( m + \frac{I_f}{r_i^2} + \frac{I_r}{r_r^2} \right) / (\lambda_2 r_1 - \lambda_1 r_2) W.
\]

Finally, with \(W = -mg\), this equation allows expressing the COM deceleration of the loaded MWC during the deceleration phase of a coast down test (cf. Equation (1)).

Appendix B

In order to quantify the terms that can be neglected in Equation (1), the deceleration value was computed 100,000 times from
Equation (1) and the two following equations:

\[
\gamma_{G} = -mg \left( \frac{\lambda t \, d t + \lambda f \, d f}{m + \frac{L}{r_f} + \frac{L}{r_w}} \right), \quad (B1)
\]

\[
\gamma_{G} = -g \left( \frac{\lambda t \, d t + \lambda f \, d f}{r_l \, w_b + r_w \, w_b} \right). \quad (B2)
\]

In each step, the terms used to compute the three deceleration values were randomly chosen within their respective variation range, defined from the previous values related in the literature or from the typical values measured on MWC: the RP ranged from 1 to 3 mm (Sauret et al. 2006, 2010; Cabelguen 2008); the radii of the front wheels ranged from 30 to 100 mm and those of the rear wheels ranged from 260 to 330 mm; the radii of the wheelbase ranged between 300 and 450 mm; the COM height of the loaded MWC ranged from 500 to 700 mm; the total mass ranged between 75 and 100 kg (Coutts 1991); the moments of inertia ranged between 0.005 and 0.02 kg m² for the front wheels and between 0.1 and 0.2 kg m² for the rear wheels (Coutts 1991; Sauret 2010) and the fore-and-aft mass distribution ranged between 30% and 60% of the mass distributed on the front wheels.

The decelerations computed from Equations (1), (B1) and (B2) were then compared.

The results showed around 3.5% error comparing the accelerations computed using Equation (1) and (B1) and around 3.3% error comparing Equation (1) with Equation (B2).