Keywords: Stiffness, Industrial, Robot, Identification, Clamped End-effector.

Abstract: This paper proposes two new methods for the identification of static stiffnesses of multi degrees of freedom heavy industrial robots. They are based on a locked link joint procedure obtained with an end-effector fixed to the environment. The first method requires only measurements of motor positions and motor torques data computed from motor current measurements and manufacturer's drive gains. The second one needs a torque sensor to measure the interaction wrench between the clamped end-effector and the environment. These methods are being experimentally validated and compared on the 2 first joints of a 6 degrees of freedom heavy 500Kg payload industrial Kuka KR500 robot.

1 INTRODUCTION

New applications of heavy industrial robots for performing machining operations like Friction Stir Welding process (FSW) need trajectories with a high accuracy end-effector position while significant forces are applied to the end-effector.

It is then necessary to identify accurately the stiffnesses to control and simulate precise and reliable motion. Identification of rigid robots has been widely investigated in the last decades, based on the Inverse Dynamic Identification Model and Least Squares estimation (IDIM-LS) (Hollerbach et al., 2008). Several approaches can be used to identify the dynamic parameters and joint dynamic stiffnesses. One approach is based on IDIM-LS technique and needs motor and joint positions and motor torques (Pham et al., 2001); (Janot et al., 2011). However, the joint positions are not measured on industrial robots. Another approach is a closed-loop output error method which needs to simulate the robot (Gautier et al., 2013); (Östring, 2003). Unfortunately, manufacturers do not want to give the control laws of their controllers.

Another approach avoids using any internal data (joint position, motor torque, control law) but it requires an external measurement of the position of the end-effector using very expensive laser-tracker sensor with variation of the payload while the motor positions are locked to constant values (Dumas et al., 2011); (Alici and Shirinzadeh, 2005). In (Pfeiffer and Holzl, 1995), each robot links are fixed alternately to identify the current joint stiffness with quasi-static test but this requires a different locking system for each link.

To overcome theses expensive and heavy procedures, this paper proposes two new methods based on a locked link joint procedure obtained with the end-effector clamped to the environment. The first method requires only motor positions measurement and motor torques data calculated from motor current measurement and manufacturer's drive gain data. The second one needs a torque sensor to measure the external interaction wrench between the clamped end-effector and the environment, which are calculated as external link torques using the jacobian matrix of the robot. A first validation of our methodologies are carried out on the 2 first joints of a 6 Degrees of Freedom (dof) heavy 500Kg payload industrial Kuka KR500 robot.

This paper is divided into 5 sections. Section 2 describes the modeling of serial robots with the end-effector attached to the base. Section 3 presents the usual method for dynamic identification of robots, based on IDIM-LS method. Section 4 is devoted to the modeling of the Kuka KR500 heavy industrial robot and to the identification of the first two axis of the robot. The last section gives the conclusion.
2 MODELING

2.1 Inverse Dynamic Identification Model with Fixed End-effector

The Inverse Dynamic Model (IDM) of a flexible robot calculates the motor torques and joint torques as a function of the joint and motor positions. It can be obtained from the Newton-Euler or the Lagrangian equations (Khalil and Dombre, 2002). It is given by the following relation with no constraint applied on the end-effector when the positions are quasi-constants (no frictions and inertias effects):

\[
\tau_{idm} = k(q_m - q) + \text{Offm}
\]

\[
\tau_{idm} = G(q) + J(q)^T F_r
\]

\[
\tau_{idm} = k(q_m - q)
\]

where \( q_m \), \( \dot{q}_m \) and \( \ddot{q}_m \) are respectively the (nx1) vectors of motor positions, velocities and accelerations; \( q \) is the (nx1) joint position; \( \tau_{idm} \) is the (nx1) vector of motor torques; \( \tau_{idm} \) is the (nx1) vector of joint torque; \( k \) is the (nxn) diagonal matrix of stiffness parameters; \( G(q) \) is the (nx1) vector of gravity torque; \( \text{Offm} \) is the (nx1) vector of motor current amplifier offset parameters; \( n \) is the number of moving links. All measurement and mechanical variables are given in S.I. unit in joint side.

\( F_r \) is the transpose of the jacobian matrix of the robot and \( F_e \) is the interaction wrench between the end-effector and the base:

\[
F_r = \begin{bmatrix} f_x \\ f_y \\ f_z \\ m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ f_z \\ m_x \\ m_y \\ m_z \end{bmatrix}^T
\]

The interaction force \( f \) is composed of the three forces \( f_x \), \( f_y \), and \( f_z \); and the interaction moment \( m \) is composed of the three moment \( m_x \), \( m_y \), and \( m_z \).

For a 6 dof robot with rigid links and with the end-effector fixed to the environment, the joint link position vector keep a constant value \( q_0 = q_{\text{measured}} \), where \( q_{\text{measured}} \) is the motor position measured when the motor and the joint wrench are close to zero. Then in the following, the jacobian matrix is calculated with \( q_{\text{measured}} \):

\[
J(q) = J(q_0) = J(q_{\text{measured}})
\]

The equations (1) becomes:

\[
\tau_{idm} = k(q_m - q_0) + \text{Offm}
\]

\[
\tau_{idm} = G(q_0) + J(q_0)^T F_r
\]

\[
\tau_{idm} = k(q_m - q_0)
\]

Introducing two offset parameters \( \text{offkm} \) and \( \text{offkl} \), the previous equation becomes:

\[
\tau_{idm} = k(q_m - \text{offkm})
\]

with \( \text{offkm} = \text{offm} - k q_0 \)

\[
\tau_{idm,i} = \left(J(q_0)^T F_r\right)
\]

\[
\tau_{idm,i} = k(q_m - \text{offkl})
\]

with \( \text{offkl} = -k q_0 - G(q_0) \)

The proposed model take into account only the joint stiffness. However the link stiffness are unknown. But, in practice, the stiffness identified values with our model are the addition of the actual joint stiffnesses and a part of link stiffnesses. Accordingly, the identified model is accurate with respect of the deformations of the robot.

2.2 Inverse Dynamic Identification Model for Joint Stiffness Identification with Motor Torques

Here, we especially want to identify the joint stiffness. Writing (5) for axis \( j \), the motor torque is function of the joint stiffness of axis \( j \):

\[
\tau_{idm,j} = k_j q_{mj} + \text{offkm} \]

(6)

The motor torque of joint \( j \) can be expressed linearly in relation to the set of dynamic parameters \( \chi_{stm,j} \) (Gautier and Khalil, 1990) to get the IDM:

\[
\tau_{idm,j} = \text{IDM}_{stm,j}(q_{mj}) \chi_{stm,j}
\]

(7)

Where \( \text{IDM}_{stm,j}(q_{mj}) \) is the (1xNs) Jacobian matrix of \( \tau_{idm,j} \) with respect to the (Nsx1) vector \( \chi_{stm,j} \) of the parameters.

\( \chi_{stm,j} \) is composed of parameters of axis \( j \) with fixed end-effector:

\[
\chi_{stm,j} = \begin{bmatrix} q_{mj} \\ \text{offkm} \end{bmatrix}
\]

(8)

The motor torque \( \tau_{mj} \) is computed from the a priori joint drive gain \( g_{ap,j} \) given by manufacturer's data and the motor current \( I \) for each axis \( j \):

\[
\tau_{mj} = g_{ap,j}^T I_j
\]

(9)
2.3 Inverse Dynamic Identification Model for Joint Stiffness Identification with External Wrench

Writing (4) for axis \( j \), the external torque of axis \( j \) is function of the joint stiffness of axis \( j \):

\[
\tau_{\text{shear},j} = k_j q_{mj} + \text{offset}_j
\]

The joint torque of joint \( j \) can be expressed linearly in relation to joint stiffness and offset parameters \( \chi_{\text{st}} j \):

\[
\tau_{\text{shear},j} = \begin{bmatrix} q_{mj} \\ \text{offset}_j \end{bmatrix} \chi_{\text{st}} j
\]

The external torques \( \tau \) can be calculated from the interaction wrench and the jacobian matrix (3) of the robot (Khalil and Dombre, 2002):

\[
\tau = J(q_0)^T F_r
\]

It is necessary to have good condition number for the jacobian matrix to have a good estimation of the joint torque.

3 IDIM-LS: INVERSE DYNAMIC IDENTIFICATION MODEL WITH LEAST SQUARES METHOD

Because of perturbations due to noise measurement and modeling errors, the actual torque \( \tau \) differs from \( \tau_{\text{shear}} \) by an error \( e \), such that:

\[
\tau = \tau_{\text{shear}} + e = \text{IDM}(q_m) \chi + e
\]

The vector \( \hat{\chi} \) is the least squares (LS) solution of an over determined system built from the sampling of (13), while the elastic deformations vary on the robot:

\[
Y = W \chi + \rho
\]

Where: \( Y \) is the \((rx1)\) measurement vector, \( W \) the \((rxb)\) observation matrix, and \( \rho \) is the \((rx1)\) vector of errors. The number of rows is \( r = nxn_r \), where the number of recorded samples is \( n_r \).

Standard deviations \( \sigma_{\chi} \), are estimated assuming that \( W \) is a deterministic matrix and \( \rho \), is a zero-mean additive independent Gaussian noise, with a covariance matrix \( C_{\rho} = E(\rho \rho^T) = \sigma_{\rho}^2 I \) (Gautier, 1997; Janot et al., 2014).

Where \( E \) is the expectation operator and \( I \), the \((rxr)\) identity matrix. An unbiased estimation of the standard deviation \( \sigma_{\rho} \) is the following:

\[
\hat{\sigma}_{\rho}^2 = \text{LS} Y - W \hat{\chi}^T (r - b)
\]

The covariance matrix of the estimation error is given by:

\[
C_{\hat{\chi}} = E[(\chi - \hat{\chi})(\chi - \hat{\chi})^T] = \sigma_{\rho}^2 (W^T W)^{-1}
\]

The relative standard deviation \( \%\sigma_{\hat{\chi}} \) is given by:

\[
\%\sigma_{\hat{\chi}} = 100 \frac{\sigma_{\hat{\chi}}}{\hat{\chi}} \text{ for } |\hat{\chi}| \neq 0
\]

Where \( \sigma_{\hat{\chi}}^2 = C_{\hat{\chi}}(i,i) \) is the \( i^{th} \) diagonal coefficient of \( C_{\hat{\chi}} \). Calculating the LS solution of (14) from perturbed data in \( W \) and \( Y \) may lead to bias if \( W \) is correlated to \( \rho \). Then, it is essential to filter data in \( Y \) and \( W \) before computing the LS solution. Velocities and accelerations are estimated by means of a band-pass filtering of the positions. To eliminate high frequency noises and torque ripples, a parallel decimation is performed on \( Y \) and on each column of \( W \). More details about data filtering can be found in (Gautier, 1997) and (Pham et al., 2001).

4 EXPERIMENTAL VALIDATION

4.1 Modeling of the Kuka KR500

The Kuka KR500 robot (see figure 2) has a serial structure with \( n=6 \) rotational joints. Its kinematics is defined using the Modified Denavit and Hartenberg (MDH) notation (Khalil and Kleinfinger, 1986). In this notation, the link \( j \) fixed frame is defined such that the \( z_j \) axis is taken along joint \( j \) axis and the \( x_j \) axis is along the common normal between \( z_{j-1} \) and \( z_j \) (see figure 1). \( \alpha_j \) and \( d_j \) parameterize the angle and distance between \( z_{j-1} \) and \( z_j \) along \( x_{j-1} \), respectively, whereas \( \theta_j \) and \( r_j \) parameterize the angle and distance between \( x_{j-2} \) and \( x_j \) along \( z_j \), respectively. All MDH positions are equal to the motor position \( q_{mj} \) given by the KRL controller of the Kuka KR500 except for axis 3 and 6. The geometric parameters are given in table 1 and allow to compute the \((6x6)\) jacobian matrix (3). All variables are given in SI unit in joint link side. The robot is characterized by a kinematic
coupling effect between the joint 4, 5 and 6 but his impact is negligible because the coupling coefficients are very low (Jubien and Gautier, 2013).

Table 1: MDH parameters of the KR500 robot.

<table>
<thead>
<tr>
<th>j</th>
<th>( \alpha_j )</th>
<th>( d_j )</th>
<th>( \theta_j )</th>
<th>( r_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \pi )</td>
<td>0</td>
<td>( q_{\text{rot}} )</td>
<td>( rl_1 = -1.045 \text{ m} )</td>
</tr>
<tr>
<td>2</td>
<td>( \pi/2 )</td>
<td>( d_2 = 0.500 \text{ m} )</td>
<td>( q_{\text{rot}} )</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>( d_3 = 1.300 \text{ m} )</td>
<td>( q_{\text{rot}} + \pi/2 )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>(-\pi/2)</td>
<td>( d_4 = -0.055 \text{ m} )</td>
<td>( q_{\text{rot}} )</td>
<td>( rl_4 = -1.025 \text{ m} )</td>
</tr>
<tr>
<td>5</td>
<td>( \pi/2 )</td>
<td>0</td>
<td>( q_{\text{rot}} )</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>(-\pi/2)</td>
<td>0</td>
<td>( q_{\text{rot}} - \pi/2 )</td>
<td>( rl_6 = -0.290 \text{ m} )</td>
</tr>
</tbody>
</table>

Table 2: Configurations of the KR500 robot.

<table>
<thead>
<tr>
<th>Conf. nb</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_3 )</th>
<th>( q_4 )</th>
<th>( q_5 )</th>
<th>( q_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.62</td>
<td>-0.89</td>
<td>1.84</td>
<td>1.62</td>
<td>(-\pi/2)</td>
<td>0.46</td>
</tr>
<tr>
<td>2</td>
<td>1.48</td>
<td>-0.87</td>
<td>1.37</td>
<td>( \pi )</td>
<td>(-1.05)</td>
<td>0</td>
</tr>
</tbody>
</table>

The first configuration allows identifying the joint stiffness of axis 1. The second (see figure 2) configuration allows identifying the joint stiffness of axis 2. Thus, it is possible to identify joint stiffness parameters without the measurement of all components of the wrench (except for axis 6).

4.3 Acquisition, Trajectories and Filtering

The sampling acquisition frequency of force sensor is 83.3(Hz) and the sampling acquisition frequency of motor currents and motor positions is 500(Hz). All the measurements are synchronized off-line. The motor currents and motor positions are given by 'Scope' function of Kuka KRC controller.

For each configuration, the end-effector of the robot is brought into contact with the rigid environment with position control. After that, the robot is controlled with force control and different levels of force on \( z_6 \) are applied on the end-effector to have a significant number of points. The end-effector is blocked only on the same direction \( z_6 \), accordingly the component \( f_z \) can be only negative. The motor positions, currents, external torques and measurement of \( f_z \) for the configurations 1 are given in figure 3. \( f_z \) cannot be measured all the time due to restriction on acquisition system so only the available measurements are showed for \( f_z \) and the external torques.

For the identifications, the cut-off frequency of the Butterworth filter is fixed to 10Hz and the cut-off frequency of decimate filter is fixed to 5Hz.

4.4 Identification of Joint Stiffness with Motor Currents

The identification of the joint stiffness of axis 1 and 2 are performed with motor currents and motor positions. The IDIM (8) is used and the motor
torques are computed with (9). The parameters of each axis are identified separately. The identified values of joint stiffness and offset are given in table 3 and 4.

<table>
<thead>
<tr>
<th>Par.</th>
<th>̂χ</th>
<th>%σ ̂χ</th>
</tr>
</thead>
<tbody>
<tr>
<td>k₁</td>
<td>6.09 10⁶</td>
<td>2.1</td>
</tr>
<tr>
<td>Offkm₁</td>
<td>-9.90 10⁶</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.82%</td>
</tr>
</tbody>
</table>

Table 3: Identified stiffness value with motor torques for axis 1.

<table>
<thead>
<tr>
<th>Par.</th>
<th>̂χ</th>
<th>%σ ̂χ</th>
</tr>
</thead>
<tbody>
<tr>
<td>k₂</td>
<td>7.81 10⁴</td>
<td>0.98</td>
</tr>
<tr>
<td>Offkm₂</td>
<td>6.76 10⁴</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.13%</td>
</tr>
</tbody>
</table>

Table 4: Identified stiffness value with motor torques for axis 2.

The stiffness parameters are well identified with a low relative standard deviation. The motor torque (circle) relative to motor position for axis 1 and 2 is showed in figure 4. A green line ( ̂k₁q₁ + Offkm₁ ) is drawn for comparison.

Figure 3: Motor position, motor current and joint torque for axis 1 and force (configuration 1).

Figure 4: Variation of motor torque relative to variation of motor position for axis 1 and 2.

4.5 Identification of Joint Stiffness with Wrench

The identification of the joint stiffness of axis 1 and 2 is performed with wrench and motor positions. The IDIM (11) is used and the joint torques are computed with (12). The jacobian matrix is computed from motor positions and with DHM parameters given in table 1. The parameters of each axis are identified separately. The identified values of joint stiffness and offset are given in table 5 and 6.

<table>
<thead>
<tr>
<th>Par.</th>
<th>̂χ</th>
<th>%σ ̂χ</th>
</tr>
</thead>
<tbody>
<tr>
<td>k₁</td>
<td>6.93 10⁴</td>
<td>0.77</td>
</tr>
<tr>
<td>Offkl₁</td>
<td>-1.12 10⁷</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.44%</td>
</tr>
</tbody>
</table>

Table 5: Identified stiffness value with external torques for axis 1.

<table>
<thead>
<tr>
<th>Par.</th>
<th>̂χ</th>
<th>%σ ̂χ</th>
</tr>
</thead>
<tbody>
<tr>
<td>k₂</td>
<td>7.81 10⁴</td>
<td>0.98</td>
</tr>
<tr>
<td>Offkl₂</td>
<td>6.76 10⁴</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.13%</td>
</tr>
</tbody>
</table>

Table 6: Identified stiffness value with external torques for axis 2.

As above, the stiffness parameters are well identified with a low relative standard deviation.
The external torque (circle) relative to motor position for axis 1 and 2 is showed in figure 5. A green line ($\hat{k}_j q_{mj} + \hat{O}f(kl_j)$) is drawn for comparison.

Figure 5: Variation of external torque relative to variation of motor position for axis 1 and 2.

### 4.6 Comparison and Discussion

The relative differences between the identified stiffness values with motor currents and wrench are given in table 7. The two methods work well to identify the joint stiffness parameters and give similar results. The differences may come from the precision of the a priori joint drive gain in (9), the precision of the ATI sensor, the precision of the MDH parameters and the approximation $J(q)\approx J(q_{m0})$.

Table 7: Relative errors between identified stiffness parameters with motor and external torques.

<table>
<thead>
<tr>
<th>Par.</th>
<th>%e</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>12.1%</td>
</tr>
<tr>
<td>$k_2$</td>
<td>2.43%</td>
</tr>
</tbody>
</table>

The major advantage of our methods is that they avoid the measurement of joint positions and/or the attitude measurement of end-effector and the use of additional payload. The position measurement of end-effector requires very expensive external measurement systems. Here, it is possible to identify the joint stiffness with and without the measurement of the wrench.

The measurement of the wrench needs a force sensor. Some robots are already equipped with this sensor to perform force control otherwise it must be installed. But our approach allows avoiding the use of wrench using only motor current if using of force sensor is too difficult.

We recall that in practice, the identified stiffness values with our model are the addition of the actual joint stiffnesses and some of the link stiffnesses.

To achieve this it is needed to clamp the end-effector to the environment which is easy if you use a beam, the soil or a vise on a production line or in a laboratory.

Unlike the method proposed in (Dumas et al., 2011), our method avoids optimizing the robot configuration to minimize its complementary stiffness matrix since the end-effector is clamped (i.e. it does not move). If only the motor torques are used, the computation of jacobian matrix and its condition number are no longer necessary.

The experimental validation on the two first axis of an industrial robot shows the effectiveness of our method.

### 5 CONCLUSION

This paper shows that it is possible to accurately identify static joint stiffnesses of industrial robots with a low-cost and easy to use procedure based on clamping the end-effector to the environment. The strong result is that the identification using only internal measurements of motor positions and torques gives similar results to those obtained with force sensor measurements of the interaction wrench. The method can be carried out on industrial robots without the need of any external sensor like expensive laser tracker or force sensor. It is a first validation of our methodologies on a simple case. Future works concern the identifications of all joint stiffness of the robot and a comparison with other joint stiffness identification methods.

### ACKNOWLEDGEMENTS

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### REFERENCES


