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Method for the Simulation of Structures and Processes - 2011

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Natural Element Method for the Simulation of Structures and Processes

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Contents

Foreword ................................................................. ix

Acknowledgements .................................................... xi

Chapter 1. Introduction .................................................. 1
  1.1. SPH method ..................................................... 3
  1.2. RKPM method .................................................. 5
    1.2.1. Conditions of reproduction ............................. 6
    1.2.2. Correction of the kernel ............................... 7
    1.2.3. Discrete form of the approximation .................. 9
  1.3. MLS based approximations .................................... 10
  1.4. Final note ..................................................... 11

Chapter 2. Basics of the Natural Element Method .................... 13
  2.1. Introduction .................................................. 13
  2.2. Natural neighbor Galerkin methods ........................ 14
    2.2.1. Interpolation of natural neighbors .................... 14
    2.2.2. Discretization ......................................... 18
    2.2.3. Properties of the interpolant based on natural neighbors .. 19
  2.3. Exact imposition of the essential boundary conditions .... 22
    2.3.1. Introduction to alpha shapes ........................ 23
    2.3.2. CNEM approaches ...................................... 25
  2.4. Mixed approximations of natural neighbor type ............ 27
    2.4.1. Considering the restriction of incompressibility ..... 28
    2.4.2. Mixed approximations in the Galerkin method ........ 32
    2.4.3. Natural neighbor partition of unity .................. 33
      2.4.3.1. Partition of unity method ........................ 33
      2.4.3.2. Enrichment of the natural neighbor interpolants .... 35
2.5. High order natural neighbor interpolants

2.5.1. Hiyoshi–Sugihara interpolant

2.5.2. The De Boor algorithm for B-splines

2.5.3. B-spline surfaces and natural neighboring

2.5.3.1. Some definitions

2.5.3.2. Surface properties

2.5.3.3. The case of repeated nodes

Chapter 3. Numerical Aspects

3.1. Searching for natural neighbors

3.2. Calculation of NEM shape functions of the Sibson type

3.2.1. Stage-1: insertion of point $x$ in the existing constrained Voronoi diagram (CVD)

3.2.1.1. Look for a tetrahedron which contains point $x$

3.2.1.2. Note concerning the problem of flat tetrahedrons

3.2.2. Stage-2: calculation of the volume measurement common to $\hat{c}_x$ and $c_0$

3.2.2.1. By the recursive Lasserre algorithm

3.2.2.2. By means of a complementary volume

3.2.2.3. By topological approach based on the CVD

3.2.2.4. By topological approach based on the Constrained Delaunay tetrahedization (CDT)

3.2.2.5. Using the Watson algorithm

3.2.3. Comparative test of the various algorithms

3.3. Numerical integration

3.3.1. Decomposition of shape function supports

3.3.2. Stabilized nodal integration

3.3.3. Discussion in connection with various quadratures

3.3.3.1. 2D patch test with a technique of decomposition of shape function supports

3.3.3.2. 2D patch test with stabilized nodal integration

3.3.3.3. 3D patch tests

3.4. NEM on an octree structure

3.4.1. Structure of the data

3.4.1.1. Description of the geometry

3.4.1.2. Interpolation on a quadtree

3.4.1.3. Numerical integration

3.4.2. Application of the boundary conditions – interface conditions

3.4.2.1. Dirichlet-type boundary conditions: use of $\hat{R}$-functions

3.4.2.2. Neumann-type boundary conditions

3.4.2.3. Partition of unity method
Chapter 4. Applications in the Mechanics of Structures and Processes 93

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1.</td>
<td>Two- and three-dimensional elasticity</td>
<td>93</td>
</tr>
<tr>
<td>4.2.</td>
<td>Indicators and estimators of error: adaptivity</td>
<td>96</td>
</tr>
<tr>
<td>4.2.1.</td>
<td>Meshless methods and adaptation</td>
<td>96</td>
</tr>
<tr>
<td>4.2.2.</td>
<td>Methodology of adaptive refinement for linear elasticity in statics</td>
<td>98</td>
</tr>
<tr>
<td>4.2.2.1.</td>
<td>Formulation in static linear elasticity</td>
<td>99</td>
</tr>
<tr>
<td>4.2.2.2.</td>
<td>The first indicator based on the rebuilding of discontinuous stress fields</td>
<td>102</td>
</tr>
<tr>
<td>4.2.2.3.</td>
<td>The second indicator based on the rebuilding of continuous fields of strain</td>
<td>103</td>
</tr>
<tr>
<td>4.2.2.4.</td>
<td>Refinement strategy based on Voronoi cells</td>
<td>105</td>
</tr>
<tr>
<td>4.3.</td>
<td>Metal extrusion</td>
<td>107</td>
</tr>
<tr>
<td>4.3.1.</td>
<td>Viscoplastic model for the extrusion of aluminum</td>
<td>108</td>
</tr>
<tr>
<td>4.3.2.</td>
<td>3D simulation of the extrusion of a cross-shaped profile</td>
<td>111</td>
</tr>
<tr>
<td>4.4.</td>
<td>Friction stir welding</td>
<td>113</td>
</tr>
<tr>
<td>4.4.1.</td>
<td>Constitutive equation</td>
<td>117</td>
</tr>
<tr>
<td>4.4.1.1.</td>
<td>Mechanical model</td>
<td>118</td>
</tr>
<tr>
<td>4.4.2.</td>
<td>Numerical results</td>
<td>120</td>
</tr>
<tr>
<td>4.5.</td>
<td>Models and numerical treatment of the phase transition: foundry and treatment of surfaces</td>
<td>123</td>
</tr>
<tr>
<td>4.5.1.</td>
<td>Introducing motion discontinuity</td>
<td>123</td>
</tr>
<tr>
<td>4.5.1.1.</td>
<td>Formulation of the thermal problem with phase transition</td>
<td>124</td>
</tr>
<tr>
<td>4.5.1.2.</td>
<td>CNEM discretization</td>
<td>126</td>
</tr>
<tr>
<td>4.6.</td>
<td>Adiabatic shearing, cutting, and high speed blanking</td>
<td>136</td>
</tr>
<tr>
<td>4.6.1.</td>
<td>General context of the implementation of the large transformations</td>
<td>140</td>
</tr>
<tr>
<td>4.6.1.1.</td>
<td>Updated Lagrangian formulation</td>
<td>141</td>
</tr>
<tr>
<td>4.6.1.2.</td>
<td>Processing the additional points</td>
<td>141</td>
</tr>
<tr>
<td>4.6.1.3.</td>
<td>Time integration scheme</td>
<td>142</td>
</tr>
<tr>
<td>4.6.1.4.</td>
<td>Processing of the contact</td>
<td>142</td>
</tr>
<tr>
<td>4.6.1.5.</td>
<td>Integration of the constitutive equation</td>
<td>142</td>
</tr>
<tr>
<td>4.6.2.</td>
<td>Applications</td>
<td>145</td>
</tr>
<tr>
<td>4.6.2.1.</td>
<td>Taylor’s bar</td>
<td>145</td>
</tr>
<tr>
<td>4.6.2.2.</td>
<td>Adiabatic shearing</td>
<td>148</td>
</tr>
<tr>
<td>4.6.3.</td>
<td>Conclusions on the numerical simulation of shearing</td>
<td>158</td>
</tr>
</tbody>
</table>

Chapter 5. A Mixed Approach to the Natural Elements 159

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1.</td>
<td>Introduction</td>
<td>159</td>
</tr>
<tr>
<td>5.2.</td>
<td>The Fraeijs de Veubeke variational principle for linear elastic problems</td>
<td>161</td>
</tr>
<tr>
<td>5.3.</td>
<td>Field decomposition</td>
<td>164</td>
</tr>
</tbody>
</table>
5.4. Discretization .................................................. 166
5.5. Discretized equations ........................................... 170
5.6. Matrix solution for linear elastic problems .................. 172
5.7. Numerical integration ........................................... 176
5.8. Linear elastic patch tests ........................................ 178
5.9. Application 1: pure bending of a linear elastic beam ........ 182
5.10. Application 2: square domain with circular hole ........... 185
5.11. Mixed approach to nonlinear problems ....................... 187
5.12. Step-by-step solution of the discretized nonlinear equations 192
5.13. Example of an elastoplastic material ......................... 195
5.15. Conclusion ...................................................... 199

Chapter 6. Flow Models .............................................. 201
6.1. Natural element method in fluid mechanics: updated Lagrangian approach .................................................. 201
  6.1.1. Mechanical model of a Newtonian fluid flow ............... 201
6.2. Free and moving surfaces ......................................... 202
  6.2.1. Use of the characteristics method ........................... 204
6.3. Short-fiber suspensions flow ....................................... 206
  6.3.1. Flow kinematics .............................................. 207
  6.3.2. Coupling of a particle method with $\alpha$-NEM ............ 208
6.4. Breaking dam problem ............................................ 208
6.5. Multi-scale approaches ........................................... 209
  6.5.1. Mechanical model ............................................. 214
    6.5.1.1. FENE model ............................................ 216
    6.5.1.2. Doi–Edwards model .................................... 217
  6.5.2. Integration of the model ..................................... 218
    6.5.2.1. Functional approximation ................................ 218
    6.5.2.2. Discretization of the model ............................... 218
    6.5.2.3. Resolution algorithm ...................................... 219
  6.5.3. Some results .................................................. 220
    6.5.3.1. Startup of a simple shear flow of an FENE fluid model 220
    6.5.3.2. Flow from an extrusion die and FENE-fluid ............ 220
    6.5.3.3. Startup of a simple shear flow of a Doi–Edwards fluid model .................................................. 223

Chapter 7. Conclusion ................................................. 225

Bibliography .......................................................... 227

Index ................................................................. 239