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## MULTI-STEP HOMOGENIZATION OF THERMOELASTIC MULTI-SCALE TUBES WITH WAVY LAYERS

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### ABSTRACT

In this paper we present a multi-step homogenization scheme of a tube made of numerous wavy cylindrical layers exhibiting periodicity with respect to both the radial and the angular direction. The proposed homogenization is a combination of successive semi-analytical, cell-problem-based, homogenization steps and a possible, micromechanics-based, homogenization in the interior of every layer. Every step of cell-problem based homogenization gives analytical expressions for the homogenized stiffness, thermal expansion and thermal conductivity.

### 1. INTRODUCTION

Shell structures are present in nature and in engineered systems ([1],[2],[3],[7]). More specifically, in the latter case they can have the form of laminate shell composites, of multilayer thermosensitive tubes, of carbon-nanotube-polymer nanocomposites, of fuzzy fibers based on multi-walled carbon nanotubes. Multi-functionality, guaranteeing desirable electronic, optical, magnetic properties, is an important property of multiscale composites having a bio-inspired microstructure constructed from the finer to the coarser scale.

On the other hand, waviness is a designed or accidental property of wavy layer materials, being intensively studied during the last decade for its strengthening or weakening effect in thin metallic and ceramic multilayers or for its magnetic and optoelectronic effect in high-speed technology, or for its isolating effect in the space laser and anti-missile programs ([4], [5],[6],[8]).

In [9], the combination of the above two microstructure geometries, i.e. shell composite structure and waviness, is considered and the homogenization of multilayer wavy tubes made of elastic materials with highly contrasted mechanical and thermal properties is proposed. In this communication we will present the principal lines of this method, as well as a numerical example of a bimetallic star-shaped composite.

### 2. PRINCIPAL LINES OF THE PROPOSED METHOD

The homogenization scheme is based on a multiscale method presented in [9]. This method is designed to furnish the effective thermoelastic properties of a star-shaped tube made of numerous thin elastic wavy layers (see fig. 1).

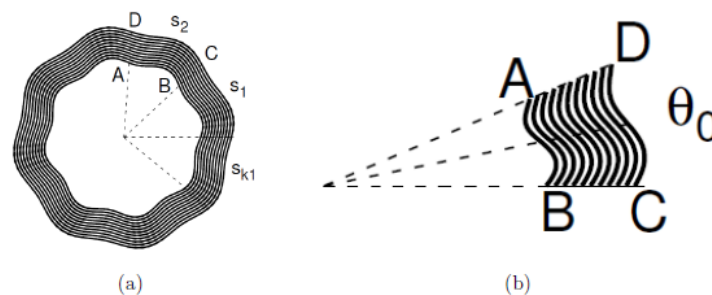


Figure 1. (a) Cross section of the tube divided into  $k_1$  sectors and (b) a typical sector.

The structure exhibits periodicity with respect to the radial and the angular direction, but in two, highly different, scales: the microstructure is much more fine with respect to the radial direction than with respect to the angular direction. This allows for applying a two-step homogenization, consisting from a first step with respect to the radial direction, giving a heterogeneous material with angular periodicity, and from a second step with respect to the angular direction, giving the effective properties. In both steps, the method gives analytical forms of the corresponding cell equations, micro-displacement gradients and homogenized thermo-elastic coefficients.

The heterogeneous problem, characterized by heterogeneous parameters dependent on a small  $\varepsilon$  tending to zero, is described in polar coordinates by the equilibrium equations

$$L_j \sigma_{rj}^\varepsilon + \frac{\sigma_{rr}^\varepsilon - \sigma_{\theta\theta}^\varepsilon}{r} + f_r = 0, \quad (1)$$

$$L_j \sigma_{\theta j}^\varepsilon + 2 \frac{\sigma_{r\theta}^\varepsilon}{r} + f_\theta = 0, \quad (2)$$

$$L_j \sigma_{zj}^\varepsilon + \frac{\sigma_{rz}^\varepsilon}{r} + f_z = 0, \quad (3)$$

the energy equation

$$L_i q_i^\varepsilon + \frac{q_r}{r} = Q, \quad (5)$$

the constitutive equations

$$\sigma_{ij}^\varepsilon = C_{ijkl}^\varepsilon \left[ \varepsilon_{kl}^\varepsilon - \alpha_{kl}^\varepsilon (T^\varepsilon - T_{ref}) \right], \quad (6)$$

$$q = -\kappa_{ij}^\varepsilon L_j T^\varepsilon, \quad (7)$$

and the strain-displacement relations

$$\varepsilon_{rr}^\varepsilon = L_r u_r^\varepsilon, \varepsilon_{\theta\theta}^\varepsilon = L_\theta u_\theta^\varepsilon + \frac{u_r^\varepsilon}{r}, \varepsilon_{zz}^\varepsilon = L_z u_z^\varepsilon, \quad (8)$$

$$\varepsilon_{\theta z}^\varepsilon = \frac{1}{2} (L_\theta u_z^\varepsilon + L_z u_\theta^\varepsilon), \varepsilon_{rz}^\varepsilon = \frac{1}{2} (L_r u_z^\varepsilon + L_z u_r^\varepsilon), \varepsilon_{r\theta}^\varepsilon = \frac{1}{2} (L_\theta u_r^\varepsilon + L_r u_\theta^\varepsilon)$$

supplemented by Cauchy, Dirichlet or mixed boundary conditions, where  $u_i$ ,  $\sigma_{ij}$ ,  $\varepsilon_{ij}$ ,  $T$ ,  $Q$ ,  $q$ , denote the displacement vector, the stress tensor, the strain tensor, the temperature, the radiation and the heat flux, respectively, while  $C_{ijkl}$ ,  $\alpha_{ij}$ ,  $\kappa_{ij}$  denote the elasticity tensor, the thermal expansion tensor and the thermal conductivity tensor, respectively. Moreover, the operators in (1), (2), (3) are defined by

$$L_r = \frac{\partial}{\partial r}, L_\theta = \frac{1}{r} \frac{\partial}{\partial \theta}, L_z = \frac{\partial}{\partial z}, \quad (9)$$

$$P_1 = \nabla_r \rho, P_2 = \frac{1}{r} \nabla_\theta \rho. \quad (10)$$

All thermomechanical functions, unknown and given, are assumed to depend on the macrocoordinates  $r$ ,  $\theta$  and additionally, in a periodic way, on a generalized periodicity surface  $\rho$ , dependent on  $\varepsilon$  (for the definition of generalized periodicity in multilayer materials see [8]). In [9] a two-step homogenization scheme is presented, based on the assumption of different scales described above.

## 2.1 Homogenization with respect to the radial periodicity $\rho_1$

In the first step of homogenization, a function  $\varphi^\varepsilon$  is expressed a function  $\varphi(r, \theta, \bar{\rho})$  of the polar coordinates  $(r, \theta)$  and  $\bar{\rho}$  bar, where  $\bar{\rho}$  bar is given by

$$\bar{\rho} = \frac{\rho}{\varepsilon}. \quad (11)$$

Then,  $\varphi$  is assumed of the form

$$\varphi^\varepsilon = \varphi^{(0)}(r, \theta, \bar{\rho}) + \varepsilon \varphi^{(1)}(r, \theta, \bar{\rho}) + \varepsilon^2 \varphi^{(2)}(r, \theta, \bar{\rho}) + \dots, \quad (12)$$

where all  $\varphi^{(i)}$  are periodic with respect to  $\bar{\rho}$ . By replacing in the equilibrium equation and in the energy equation one obtains from the condition of zero coefficients for  $\varepsilon^{-2}$  and  $\varepsilon^{-1}$  the cell problem

$$\bar{L}_j^{(1)} C_{ijmn} + \bar{L}_j^{(1)} \left( C_{ijkl} \bar{L}_k^{(1)} N_l^{mn(1)} \right) = 0 \quad (13)$$

$$\bar{L}_j^{(1)} C_{ijkl} \alpha_{kl} + \bar{L}_j^{(1)} C_{ijkl} \bar{L}_k^{(1)} N_l^{0(1)} = 0 \quad (14)$$

$$\bar{L}_i^{(1)} \kappa_{im} + \bar{L}_j^{(1)} \kappa_{ij} \bar{L}_j^{(1)} W^{m(1)} = 0 \quad (15)$$

where  $N_l^{mn(1)}$ ,  $N_l^{0(1)}$ ,  $W^{m(1)}$  are the unknown fluctuating functions, periodic in  $\bar{\rho}$  and satisfying the continuity conditions

$$\left[ | N_i^{mn(1)} | \right] = 0, \left[ | \left( C_{ijmn} + C_{ijkl} \bar{L}_k^{(1)} N_l^{mn(1)} \right) n_j | \right] = 0, \quad (16)$$

$$\left[ | N_i^{0(1)} | \right] = 0, \left[ | \left( C_{ijmn} \alpha_{kl} + C_{ijkl} \bar{L}_k^{(1)} N_l^{0(1)} \right) n_j | \right] = 0, \quad (17)$$

$$\left[ | W_m^{(1)} | \right] = 0, \left[ | \left( \kappa_{im} + \kappa_{ij} \bar{L}_j^{(1)} W^{m(1)} \right) n_i | \right] = 0, \quad (18)$$

and where  $\bar{L}_i^{(1)}$  are given by

$$\bar{L}_r^{(1)} = \nabla_r \rho \frac{\partial}{\partial \bar{\rho}}, \bar{L}_\theta^{(1)} = \frac{1}{r} \nabla_\theta \rho \frac{\partial}{\partial \bar{\rho}}, \bar{L}_z^{(1)} = 0. \quad (19)$$

The solution of the cell problem is of the form

$$\frac{\partial N_1^{\alpha(1)}}{\partial \bar{\rho}} = \frac{B_2^{(1)} \left( \lambda_1^{\alpha(1)} - P_1^{(1)} C_{6\alpha} - P_2^{(1)} C_{2\alpha} \right) - A_2^{(1)} \left( \lambda_2^{\alpha(1)} - P_1^{(1)} C_{1\alpha} - P_2^{(1)} C_{2\alpha} \right)}{A_1^{(1)} B_2^{(1)} - A_2^{(1)} B_1^{(1)}}, \quad (20)$$

$$\frac{\partial N_2^{\alpha(1)}}{\partial \bar{\rho}} = \frac{A_1^{(1)} \left( \lambda_2^{\alpha(1)} - P_1^{(1)} C_{1\alpha} - P_2^{(1)} C_{2\alpha} \right) - B_1^{(1)} \left( \lambda_1^{\alpha(1)} - P_1^{(1)} C_{6\alpha} - P_2^{(1)} C_{2\alpha} \right)}{A_1^{(1)} B_2^{(1)} - A_2^{(1)} B_1^{(1)}}, \quad (21)$$

$$\frac{\partial N_3^{\alpha(1)}}{\partial \bar{\rho}} = \frac{\lambda_3^{\alpha(1)} - P_1^{(1)} C_{5\alpha} - P_2^{(1)} C_{4\alpha}}{P_1^{(1)} C_{55} P_1^{(1)} + P_2^{(1)} C_{55} P_2^{(1)}}, \quad (22)$$

$$\frac{\partial N_1^{0(1)}}{\partial \bar{\rho}} = \frac{B_2^{(1)}(\bar{\lambda}_1^{\alpha(1)} - A_3^{(1)}) - A_2^{(1)}(\bar{\lambda}_2^{\alpha(1)} - B_3^{(1)})}{A_1^{(1)}B_2^{(1)} - A_2^{(1)}B_1^{(1)}}, \quad (23)$$

$$\frac{\partial N_2^{0(1)}}{\partial \bar{\rho}} = \frac{A_1^{(1)}(\bar{\lambda}_1^{\alpha(1)} - B_3^{(1)}) - B_1^{(1)}(\bar{\lambda}_1^{\alpha(1)} - A_3^{(1)})}{A_1^{(1)}B_2^{(1)} - A_2^{(1)}B_1^{(1)}}, \quad (24)$$

$$\frac{\partial N_3^{0(1)}}{\partial \bar{\rho}} = \frac{\bar{\lambda}_3^{\alpha(1)} - D_2^{(1)}}{D_1^{(1)}}, \quad (25)$$

$$\frac{\partial W_1^{\alpha(2)}}{\partial \bar{\theta}} = \frac{\tilde{\lambda}_1^{\alpha(1)} - F_1^{(1)}}{F_0^{(1)}}, \quad (26)$$

$$\frac{\partial W_2^{\alpha(2)}}{\partial \bar{\theta}} = \frac{\tilde{\lambda}_2^{\alpha(1)} - F_2^{(1)}}{F_0^{(1)}}, \quad (27)$$

$$\frac{\partial W_3^{\alpha(2)}}{\partial \bar{\theta}} = \frac{\tilde{\lambda}_3^{\alpha(1)} - F_3^{(1)}}{F_0^{(1)}}, \quad (28)$$

$a = 1, 2, 3, 4, 5, 6$ , where the analytical form of all coefficients ( $A_1^{(1)}, A_2^{(1)}, A_3^{(1)}, B_1^{(1)}, B_2^{(1)}, B_3^{(1)}, D_1^{(1)}, D_2^{(1)}, F_0^{(1)}, F_1^{(1)}, F_2^{(1)}, F_3^{(1)}, \lambda_1^{\alpha(1)}, \lambda_2^{\alpha(1)}, \lambda_3^{\alpha(1)}, \bar{\lambda}_1^{\alpha(1)}, \bar{\lambda}_2^{\beta(1)}, \bar{\lambda}_3^{\alpha(1)}, \tilde{\lambda}_1^{\alpha(1)}, \tilde{\lambda}_2^{\alpha(1)}, \tilde{\lambda}_3^{\alpha(1)}$ ) are given in [9]. The above coefficients depend on the thermomechanical properties of the constituents and their volume fractions and on the gradient of the wavy walls.

The above solutions allow one to define the homogenized coefficients resulting from the first step of homogenization,

$$\begin{aligned} C_{ba}^{eff(1)} = & \langle C_{ba} + C_{b1}\bar{L}_r^{(1)}N_r^a + C_{b6}\bar{L}_r^{(1)}N_\theta^a + C_{b5}\bar{L}_r^{(1)}N_z^a + \\ & + C_{b6}\bar{L}_\theta^{(1)}N_r^a + C_{b2}\bar{L}_\theta^{(1)}N_\theta^a + C_{b4}\bar{L}_\theta^{(1)}N_z^a \rangle, \end{aligned} \quad (29)$$

$$\begin{aligned} \alpha_b^{eff(1)} = & \left( C_{ba}^{eff(1)} \right)^{-1} \langle C_{a1}\alpha_1 + C_{a2}\alpha_2 + C_{a3}\alpha_3 + (C_{11}\alpha_1 + C_{12}\alpha_2 + C_{13}\alpha_3)\bar{L}_r^{(1)}N_r^{\alpha(1)} + \\ & + (C_{21}\alpha_1 + C_{22}\alpha_2 + C_{23}\alpha_3)\bar{L}_\theta^{(1)}N_\theta^{\alpha(1)} \rangle, \end{aligned} \quad (30)$$

$$\kappa_{im}^{eff(1)} = \langle \kappa_{im} + \kappa_{ir}\bar{L}_r^{(1)}W_r^{(1)} + \kappa_{i\theta}\bar{L}_r^{(1)}W_\theta^{(1)} \rangle, \quad (31)$$

where  $a, b = 1-6$ ,  $l, m = r, \theta, z$  and  $\langle f \rangle$  denotes the mean value of  $f$  over the unit cell. The above coefficients are used as input data for the second step of homogenization.

## 2.2 Second step of homogenization with respect to the angular direction $\theta$

The second step of homogenization with respect to the angular direction, in which all functions  $\varphi^\varepsilon$  are assumed to depend on  $\theta$  and  $\bar{\theta}$ , where  $\bar{\theta} = \frac{\theta}{\varepsilon}$ , leads to a second one-dimensional cell problem

$$\bar{L}_\theta^{(2)}C_{i\theta mn}^{eff(1)} + \bar{L}_\theta^{(2)}\left(C_{i\theta kl}^{eff(1)}\bar{L}_k^{(2)}N_l^{mn(2)}\right) = 0, \quad (32)$$

$$\bar{L}_j^{(2)} C_{ijkl}^{eff(1)} \alpha_{kl}^{eff(1)} + \bar{L}_j^{(2)} C_{ijkl}^{eff(1)} \bar{L}_k^{(2)} N_l^{0(2)} = 0, \quad (33)$$

$$\bar{L}_i^{(2)} \kappa_{im}^{eff(1)} + \bar{L}_i^{(2)} \kappa_{ij}^{eff(1)} \bar{L}_j^{(2)} W_m^{(2)} = 0, \quad (34)$$

where  $N_i^{mn(2)}$ ,  $N_i^{0(2)}$ ,  $W_m^{(2)}$  are the unknown fluctuating functions periodic in  $\bar{\rho}$  satisfying the continuity conditions

$$\left[ | N_i^{mn(2)} | \right] = 0, \left[ | \left( C_{ijmm}^{eff(1)} + C_{ijkl}^{eff(1)} \bar{L}_k^{(2)} N_l^{mn(2)} \right) n_j | \right] = 0, \quad (35)$$

$$\left[ | N_i^{0(2)} | \right] = 0, \left[ | \left( C_{ijmm}^{eff(1)} \alpha_{kl}^{eff(1)} + \left( C_{ijkl}^{eff(1)} \bar{L}_k^{(2)} N_l^{0(2)} \right) \right) n_j | \right] = 0, \quad (36)$$

$$\left[ | W_m^{(2)} | \right] = 0, \left[ | \left( \kappa_{im}^{eff(1)} + \kappa_{ij}^{eff(1)} \bar{L}_j^{(2)} W_m^{(2)} \right) n_i | \right] = 0. \quad (37)$$

The solution of the equations of the cell problem is of the form

$$\frac{\partial N_1^{\alpha(2)}}{\partial \bar{\theta}} = \frac{\left( \lambda_1^{\alpha(2)} - P_2^{(2)} C_{2\alpha}^{eff(1)} \right) C_{22}^{eff(1)} - \left( \lambda_2^{\alpha(2)} - P_2^{(2)} C_{6\alpha}^{eff(1)} \right) C_{62}^{eff(1)}}{P_2^{(2)} \left( C_{26}^{eff(1)} C_{62}^{eff(1)} - C_{66}^{eff(1)} C_{22}^{eff(1)} \right)}, \quad (39)$$

$$\frac{\partial N_2^{\alpha(2)}}{\partial \bar{\theta}} = \frac{-\left( \lambda_1^{\alpha(2)} - P_2^{(2)} C_6^{eff(1)} \right) C_{26}^{eff(1)} + \left( \lambda_2^{\alpha(2)} - P_2^{(2)} C_{2\alpha}^{eff(1)} \right) C_{66}^{eff(1)}}{P_2^{(2)} \left( C_{26}^{eff(1)} C_{62}^{eff(1)} - C_{66}^{eff(1)} C_{22}^{eff(1)} \right)}, \quad (40)$$

$$\frac{\partial N_3^{\alpha(2)}}{\partial \bar{\theta}} = \frac{\lambda_3^{\alpha(2)} - P_2^{(2)} C_{4\alpha}^{eff(1)}}{P_2^{(2)} C_{44}^{eff(1)}}, \quad (41)$$

$$\frac{\partial N_1^{0(2)}}{\partial \bar{\theta}} = \frac{B_2^{(2)} \left( \bar{\lambda}_1^{\alpha(2)} - A_3^{(2)} \right) - A_2^{(2)} \left( \bar{\lambda}_2^{\alpha(2)} - B_3^{(2)} \right)}{A_1^{(2)} B_2^{(2)} - A_2^{(2)} B_1^{(2)}}, \quad (42)$$

$$\frac{\partial N_2^{0(2)}}{\partial \bar{\theta}} = \frac{A_1^{(2)} \left( \bar{\lambda}_1^{\alpha(2)} - B_3^{(2)} \right) - B_1^{(2)} \left( \bar{\lambda}_2^{\alpha(2)} - A_3^{(2)} \right)}{A_1^{(2)} B_2^{(2)} - A_2^{(2)} B_1^{(2)}}, \quad (43)$$

$$\frac{\partial N_3^{0(2)}}{\partial \bar{\theta}} = \frac{\bar{\lambda}_3^{\alpha(2)} - D_2^{(2)}}{D_1^{(2)}}, \quad (44)$$

$$\frac{\partial W_1^{\alpha(2)}}{\partial \bar{\theta}} = \frac{\bar{\lambda}_1^{\alpha(2)} - P_2^{(2)} \kappa_{\theta r}^{eff(1)}}{P_2^{(2)} \kappa_{\theta\theta}^{eff(1)} P_2^{(2)}}, \quad (45)$$

$$\frac{\partial W_2^{\alpha(2)}}{\partial \bar{\theta}} = \frac{\bar{\lambda}_2^{\alpha(2)} - P_2^{(2)} \kappa_{\theta\theta}^{eff(1)}}{P_2^{(2)} \kappa_{\theta\theta}^{eff(1)} P_2^{(2)}}, \quad (46)$$

$$\frac{\partial W_3^{\alpha(2)}}{\partial \bar{\theta}} = \frac{\bar{\lambda}_3^{\alpha(2)} - P_2^{(2)} \kappa_{\theta z}^{eff(1)}}{P_2^{(2)} \kappa_{\theta\theta}^{eff(1)} P_2^{(2)}}, \quad (47)$$

$a = 1, 2, 3, 4, 5, 6$ , where the analytical form of all coefficients ( $A_1^{(2)}, A_2^{(2)}, A_3^{(2)}, B_1^{(2)}, B_2^{(2)}, B_3^{(2)}, D_1^{(2)}, D_2^{(2)}, \lambda_1^{\alpha(2)}, \lambda_2^{\alpha(2)}, \lambda_3^{\alpha(2)}, \bar{\lambda}_1^{\alpha(2)}, \bar{\lambda}_2^{\alpha(2)}, \bar{\lambda}_3^{\alpha(2)}, \tilde{\lambda}_1^{\alpha(2)}, \tilde{\lambda}_2^{\alpha(2)}, \tilde{\lambda}_3^{\alpha(2)}$ ) are given in [9].

### 2.3 Effective properties

The solutions of the second cell problem are used to define the effective properties of the star-shaped composite. More specifically, one obtains the effective elasticity tensor

$$C_{ba}^{eff} = \langle C_{ba}^{eff(1)} + C_{b6}^{eff(1)} \bar{L}_\theta^{(2)} N_r^a + C_{b2}^{eff(1)} \bar{L}_\theta^{(2)} N_\theta^a + C_{b4}^{eff(1)} \bar{L}_\theta^{(2)} N_z^a \rangle, \quad (49)$$

for  $a, b = 1, 2, 3, 4, 5, 6$ , the effective thermal expansion tensor

$$\begin{aligned} \alpha_b^{eff} = & \left( C_{ba}^{eff} \right)^{-1} \langle C_{a1}^{eff(1)} \alpha_1^{eff(1)} + C_{a2}^{eff(1)} \alpha_2^{eff(1)} + C_{a3}^{eff(1)} \alpha_3^{eff(1)} + 2C_{a6}^{eff(1)} \alpha_6^{eff(1)} + \\ & \left( C_{61}^{eff(1)} \alpha_1^{eff(1)} + C_{62}^{eff(1)} \alpha_2^{eff(1)} + C_{63}^{eff(1)} \alpha_3^{eff(1)} + 2C_{66}^{eff(1)} \alpha_6^{eff(1)} \right) \bar{L}_\theta^{(1)} N_r^{\alpha(2)} + \\ & + \left( C_{21}^{eff(1)} \alpha_1^{eff(1)} + C_{22}^{eff(1)} \alpha_2^{eff(1)} + C_{23}^{eff(1)} \alpha_3^{eff(1)} + 2C_{26}^{eff(1)} \alpha_6^{eff(1)} \right) \bar{L}_\theta^{(1)} N_\theta^{\alpha(2)} \rangle \end{aligned} \quad (50)$$

for  $a, b = 1, 2, 3, 4, 5, 6$  and the effective thermal conductivity tensor

$$\kappa_{im}^{eff} = \langle \kappa_{im}^{eff(1)} + \kappa_{ir}^{eff(1)} \bar{L}_r^{(2)} W_m^{(2)} + \kappa_{i\theta}^{eff(1)} \bar{L}_\theta^{(2)} W_m^{(2)} \rangle, \quad (51)$$

for  $a, b = 1, 2, 3, 4, 5, 6$ .

### 3. NUMERICAL EXAMPLE

We consider a bimetallic star-shaped shell composite having multilayered sinusoidal walls with parametric equation

$$\rho^{(1)} = r + H \sin(k_1 \theta) = k \varepsilon, \quad (52)$$

$$\rho^{(2)} = \theta = k \varepsilon^m, \quad (53)$$

for  $k = 1, 2, \dots, m < \frac{1}{2}$ , where  $k_1$  is the number of sectors of the cross-section, made of two alternative elastic isotropic materials having the properties shown in tables 1-4.

Property	Material 1 (Steel)	Material 2 (Aluminum)
Young Modulus (GPa)	206.742	72.041
Poisson Ratio	0.30	0.35
Thermal expansion coefficient (1/K)	12.265E-6	23.201E-6
Coefficient of heat conductivity (W/nk)	65.106	207.498
Volume fraction	50 %	50 %

Table 1. Thermomechanical properties of constituents and volume fraction.

Mechanical Properties	Material 1 (Steel)	Material 2 (Aluminum)
$C_{11}, C_{22}, C_{33}$	278.307	115.621
$C_{12}, C_{13}, C_{23}$	119.274	62.258
$C_{44}, C_{55}, C_{66}$	79.516	26.682
$C_{14}, C_{15}, C_{16}$	0	0
$C_{24}, C_{25}, C_{26}$	0	0
$C_{34}, C_{35}, C_{36}$	0	0

Table 2. Symmetric stiffness tensor coefficients of the two constituents (in Voigt notation and in GPa)

Properties	Material 1 (Steel)	Material 2 (Aluminum)
$\alpha_1, \alpha_2, \alpha_3$	12.265E-6	23.201E-6
$\alpha_4, \alpha_5, \alpha_6$	0	0

Table 3. Symmetric thermal expansion tensor (in 1/K) of the two constituents

Properties	Material 1 (Steel)	Material 2 (Aluminum)
$k_{rr}, k_{\theta\theta}, k_{zz}$	65.106	207.498
$k_{r\theta}, k_{rz}, k_{\theta z}$	0	0

Table 4. Symmetric thermal expansion tensor (in 1/K) of the two constituents

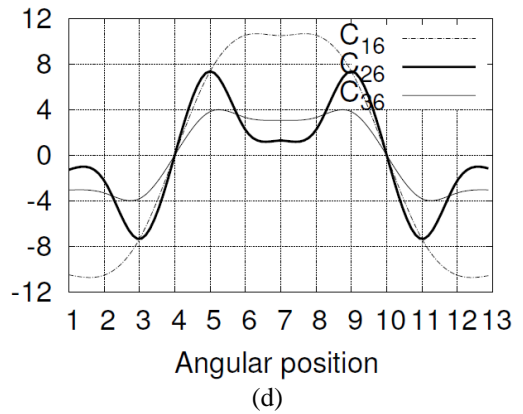
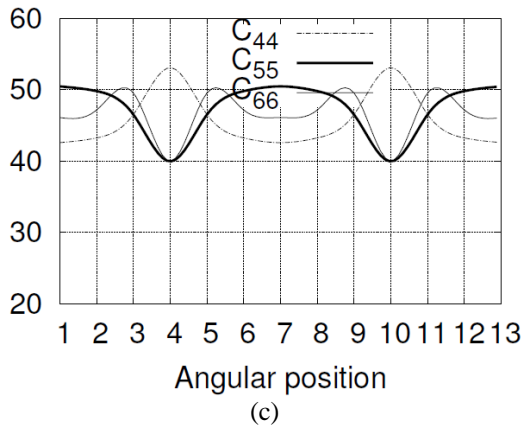
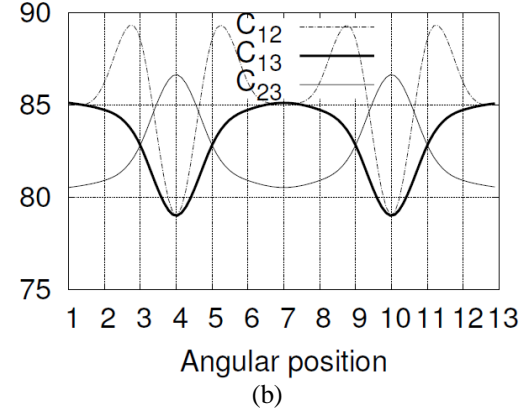
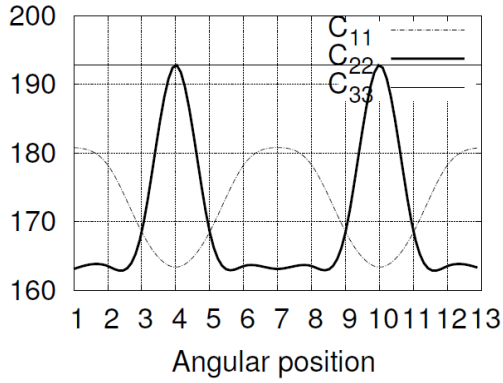
The gradients of the generalized periodicity functions for the two steps are given by

$$\nabla_r \rho^{(1)} = 1 + k_1 H \theta \cos(k_1 \theta), \quad (54)$$

$$\nabla_\theta \rho^{(1)} = k_1 H \theta \cos(k_1 \theta), \quad (55)$$

$$\nabla_r \rho^{(2)} = 0, \quad (56)$$

$$\nabla_\theta \rho^{(2)} = 1, \quad (57)$$





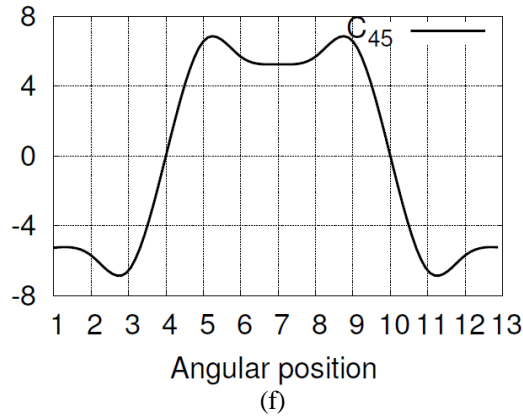


Figure 2. Variation of coefficients of the stiffness matrix (in GPa) from the 1<sup>st</sup> step of homogenization with respect to the angle  $\theta$  in inner layer.

In fig. 2, 3 and 4 we see the variation with respect to the angular direction of the coefficients of stiffness matrix, thermal expansion and thermal conductivity, respectively, deduced from the first step of homogenization. The homogenized coefficients correspond to a monoclinic material. We verify that large gradient of the wavy layers enhances shear strength in all planes (fig.2(c)) and tensile strength along the angular direction (fig.2(a)). On the other hand, it reduces the tensile strength along the radial direction. At points where the gradient of the layer is zero (points 4 and 10), all coefficients are equal to the corresponding coefficients in [8], while the homogenized coefficients  $\alpha_1$ ,  $\alpha_3$  and  $\alpha_6$  take their maximum and  $\alpha_2$  its minimum value. At points where the gradient of the layer takes its maximum value (points 1, 7 and 13) the opposite holds. In addition, the homogenized value  $\alpha_6$  from the first step of homogenization is oscillating around zero (fig. 3(d)). Analogous observations are deduced from fig. 4.

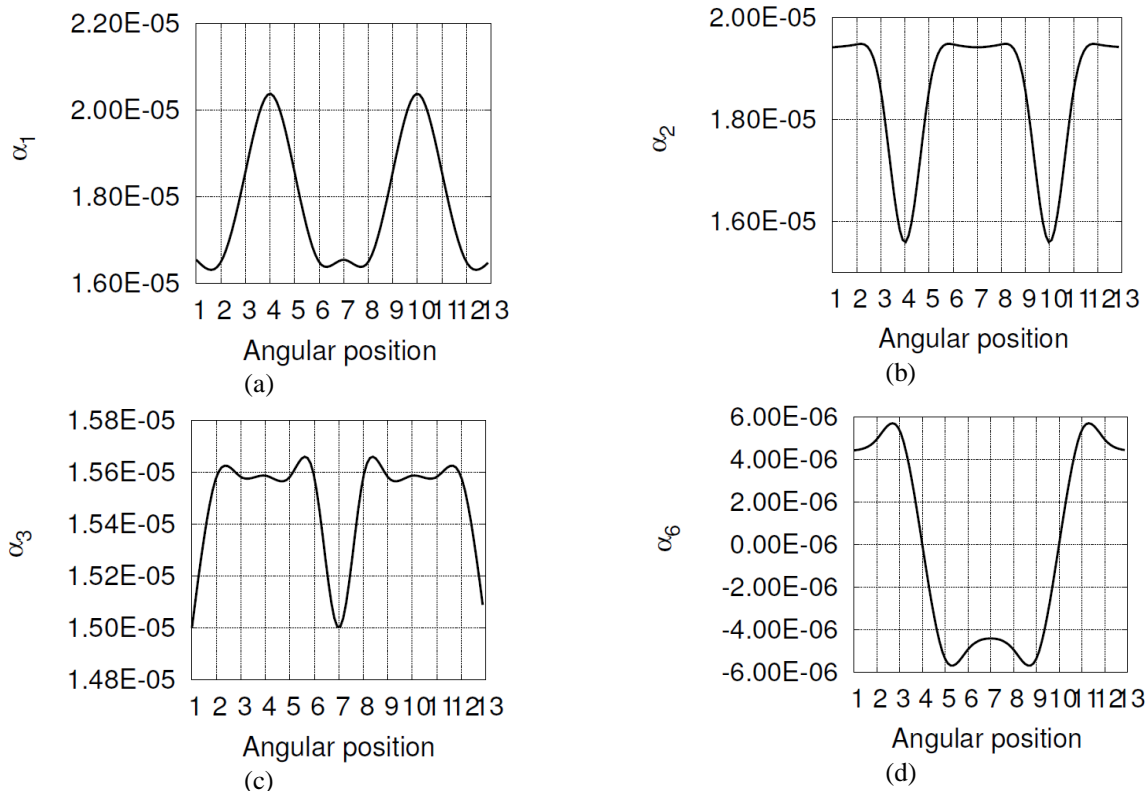


Figure 4: Variation of coefficients of the thermal expansion tensor (in 1/K) from the 1<sup>st</sup> step of homogenization in inner layer: (a)  $\alpha_1$ , (b)  $\alpha_2$ , (c)  $\alpha_3$  and (d)  $\alpha_6$ . In addition  $\alpha_4 = \alpha_5 = 0$ .

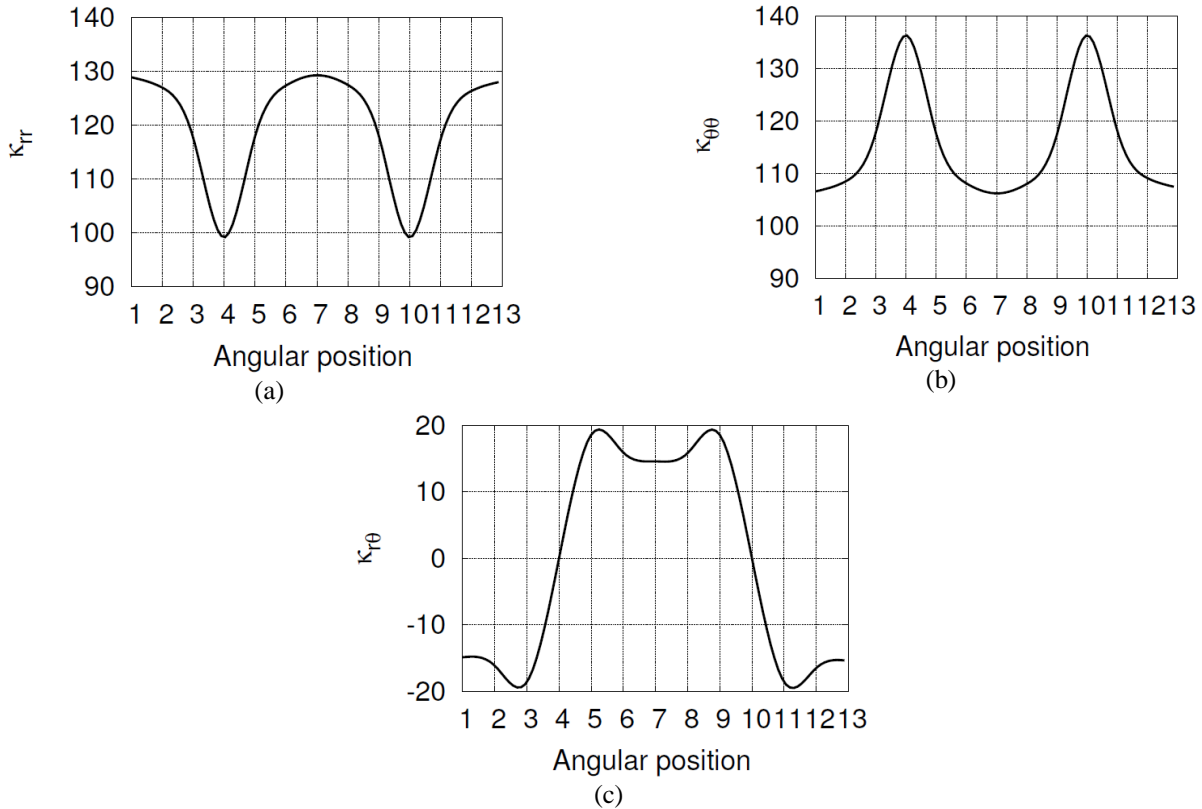


Figure 4. Variation of coefficients of the thermal conductivity tensor (in W/mK) from the 1<sup>st</sup> step of homogenization in inner layer: (a)  $\kappa_{rr}$ , (b)  $\kappa_{\theta\theta}$  and (c)  $\kappa_{r\theta}$ . In addition  $\kappa_{zz}=136.302$  W/mK and  $\kappa_{rz} = \kappa_{z\theta} = 0$ .

In table 5, the effective stiffness matrix, resulting from the 2<sup>nd</sup> step of homogenization, is shown. We verify that the overall behavior is orthotropic. From these values, we compute the effective Young moduli  $E_{rr} = 118.778$  and  $E_{\theta\theta} = 116.497$  and the effective shear modulus  $G_{r\theta} = 23.103$ , showing an important strengthening 15.6% of the shear resistance with respect to the wavyless tube, while the hoop resistance is reduced considerably (16.5%). In table 6, the effective thermal expansion tensor is shown, exhibiting a waviness-induced reduction 15% of the radial effective expansion accompanied by an enhancement 20% of the angular expansion. The inverse effect is observed in the effective conductivity behavior shown in table 7: the radial conductivity coefficient is enhanced 19%, while the angular conductivity coefficient is reduced 23% compared to the tube with circular layers.

Table 5. Effective symmetric stiffness matrix (in GPa).

$$C^{eff} = \begin{pmatrix} 171.906 & 85.154 & 82.807 & 0 & 0 & 0 \\ 85.154 & 168.541 & 81.934 & 0 & 0 & 0 \\ 82.807 & 81.934 & 192.606 & 0 & 0 & 0 \\ 0 & 0 & 0 & 45.362 & 0 & 0 \\ 0 & 0 & 0 & 0 & 46.776 & 0 \\ 0 & 0 & 0 & 0 & 0 & 46.207 \end{pmatrix}$$

Table 6. Effective thermal expansion tensor (in 1/K) in inner layer.

$$a^{eff} = \begin{pmatrix} 17.737E-6 \\ 18.582E-6 \\ 15.448E-6 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Table 7. Effective thermal conductivity tensor (in W/mK) in inner layer.

$$k^{eff} = \begin{pmatrix} 118.146 & 0 & 0 \\ 0 & 105.560 & 0 \\ 0 & 0 & 136.302 \end{pmatrix}$$

#### 4. CONCLUSIONS

The principal goal of this paper is to evaluate the waviness effect to the overall thermomechanical behavior of the star-shaped tube for any wavy form of the layers. Under the assumption of a microstructural two-scale periodicity, the two-dimensional cell problem needing a FEM-based computational homogenization is reduced to two one-dimensional cell problems, giving exact analytical expressions for the effective thermomechanical parameters. The related numerical results for a bimetallic tube with sinusoidal layers show that waviness affects in an anisotropic way considerably this behavior, by enhancing the shear resistance, the radial conductivity and the angular expansion and by reducing the hoop stress capacity, the radial expansion and the angular conductivity.

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