New method to calculate the conduction durations of the switches in a n-leg 2-level Voltage Source Inverter

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Abstract

The proposed method allows to calculate explicitly, using mathematical notions of barycenter and mixed product, the conduction duration switches of a n-leg 2-level Voltage Source Inverter (**VS**I) supplying a n-wire load. This method depends on a new geometrical and vectorial VSI characterization. Its geometrical feature enables optimization and graphic representation such as the Space Vector Method (**SVM**) for a 3-leg VSI. Its vectorial feature enables to generalize to n phase systems the properties found out with three phase systems.

Introduction

When a VSI is controlled with a carrier-based PWM, the conduction durations of the switches have to be calculated to obtain the correct average values of the voltages applied to the load. The more classical way is the Suboscillation Method [9] which determines the intersections between a triangular shape wave and the desired average values of voltages. For a 3-leg 2-level VSI supplying a star connected load with isolated neutral, another method, more favorable to optimization [2],[4] is the Space Vector Method. Then, to obtain the conduction durations, projections of the desired vectors must be achieved (Fig 1). However, this last method can hardly be generalized to the study of n-leg VSI supplying n-wire loads [13], [14] or even a 3-leg VSI supplying a star connected load with neutral not isolated [6]. It is always possible to use the Suboscillation Method but in this case we have no more geometrical tools to analyze and optimize the performances of different control laws. A few works, [1], [5], develop, for 4-leg VSI a 3-dimensional approach by using vectors that belong to a 3-dimensional space. Other works [8], [10] use more general methods for n-leg VSI but with no geometrical approach.

We propose in this paper a general geometrical vectorial characterization of VSI. So, a n-dimensional vectorial space F_n is introduced for studying a n-leg VSI. This characterization enables to use geometrical tools for optimization of n-leg VSI control as the SVM for a 3-leg VSI. Then, we describe how to determine explicitly, in a such space F_n, the conduction durations of the switches. From this point of view, it is a generalization of the way to achieve, by SVM, the conduction durations for a 3-leg VSI (Fig 1). At last, we explicit for a 3-leg 2-level VSI the relations of the method with SVM and with Suboscillation method.

Characterization of a n-leg 2-level VSI

Vectorial space F_n associated to VSI

The n-leg inverter represented in Fig 2 imposes n voltages v_{ck}. So, we associate to this converter a vectorial space $\mathbf{F_n}$ with an orthonormal base of vectors ($\mathbf{x_{c1}}$, $\mathbf{x_{c2}}$,…, $\mathbf{x_{cn}}$). We can define then a voltage vector:

$$\mathbf{v_c} = v_{c1}\,\mathbf{x_{c1}} + v_{c2}\,\mathbf{x_{c2}} + \ldots + v_{cn}\,\mathbf{x_{cn}}.$$

This voltage vector characterizes the different voltages that the inverter can impose to the load. Besides, it is easy to find out the voltage v_{ck} from $\mathbf{v_c}$: we have only to achieve the scalar product of the two vectors $\mathbf{v_c}$ and $\mathbf{x_{ck}}$: $x_{ck} = \mathbf{v_c} \cdot \mathbf{x_{ck}}$. From this point of view the vector $\mathbf{v_c}$ is more convenient than complex vector $\underline{\mathbf{vc}}$. For this last one, it is not so easy to obtain v_{ck} from $\underline{\mathbf{vc}}$.

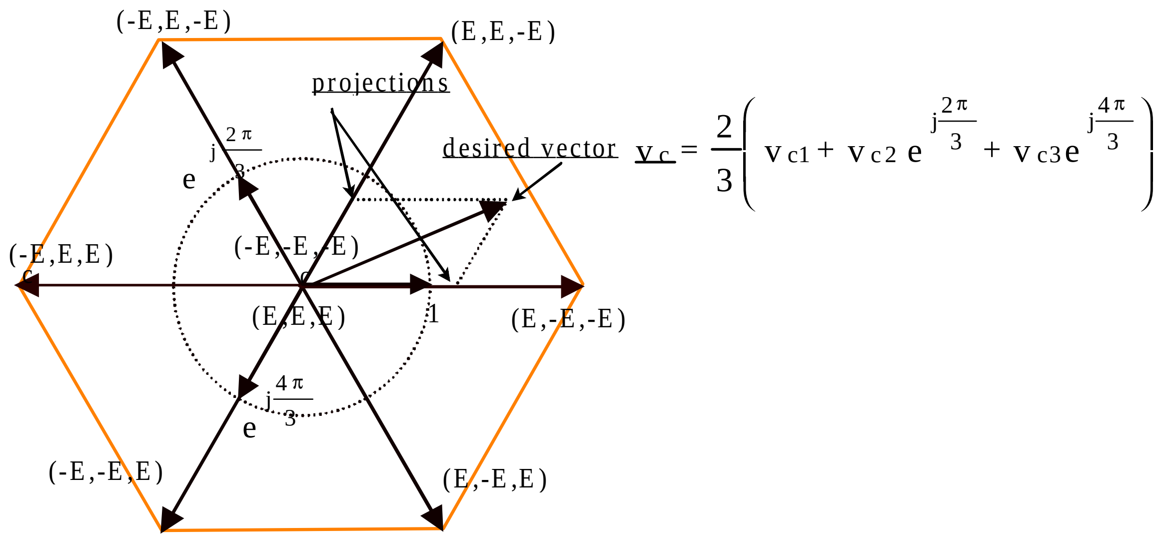

Fig 1: Characterization by Space Vector Method of a 3-leg 2-level VSI connected to a star load with isolated neutral.

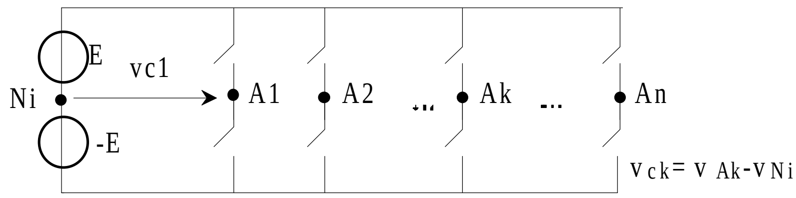

Fig 2: Representation of a n-leg 2-level inverter.

Geometrical characterization of VSI

Each coordinate of $\mathbf{v_c}$ can accept two values, +E and –E. Consequently, a family of P = 2^n vectors $\mathbf{v_{cr}}$ characterizes the inverter. Let us consider, for geometrical representation, the points O and M_r such as $\mathbf{OM_r} = \mathbf{v_{cr}}$. The P points M_r are the vertex of a polyhedron **B**.
For a 3-leg 2-level VSI (n=3), we obtain thus the 8 following vectors:

$$\begin{cases} \mathbf{OM_0} = -E\mathbf{x_{c1}} - E\mathbf{x_{c2}} - E\mathbf{x_{c3}} & \mathbf{OM_4} = -E\mathbf{x_{c1}} + E\mathbf{x_{c2}} + E\mathbf{x_{c3}} \\ \mathbf{OM_1} = +E\mathbf{x_{c1}} - E\mathbf{x_{c2}} - E\mathbf{x_{c3}} & \mathbf{OM_5} = -E\mathbf{x_{c1}} - E\mathbf{x_{c2}} + E\mathbf{x_{c3}} \\ \mathbf{OM_2} = +E\mathbf{x_{c1}} + E\mathbf{x_{c2}} - E\mathbf{x_{c3}} & \mathbf{OM_6} = +E\mathbf{x_{c1}} - E\mathbf{x_{c2}} + E\mathbf{x_{c3}} \\ \mathbf{OM_3} = -E\mathbf{x_{c1}} + E\mathbf{x_{c2}} - E\mathbf{x_{c3}} & \mathbf{OM_7} = +E\mathbf{x_{c1}} + E\mathbf{x_{c2}} + E\mathbf{x_{c3}} \end{cases}$$

In Fig 3 the polyhedron **B** is represented. It is simply a cube. We can point out that this characterization of VSI is more complete, even for n=3, than this one obtained with the SVM because no hypothesis on the connecting kind of the VSI load is putting forward. The two different points M_0 and M_7 represent two different VSI states which correspond only to the nul vector in VSM representation (Fig 1).
For a 3-level VSI, this kind of characterization can be useful for studying the ground current escaping through stray capacitors [6] but also the bearing current and shaft voltage [7] or the mean to reduce common mode emissions [3]. Elimination of common mode voltage in three phase converters by use of 4-leg VSI ([10], [11]) could also take advantage of this characterization.

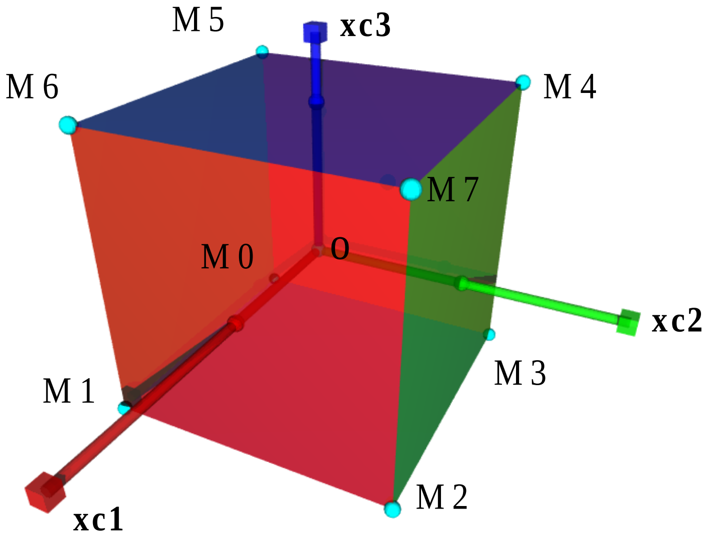

Fig 3: Characterization of a 3-leg 2-level Voltage Source Inverter (VSI)

Average control and barycenter notion

We consider a carrier-based PWM with a period carrier T. At the kT instant, $<\mathbf{v_c}>$, the mean value of the vector $\mathbf{v_c}(t)$ can be expressed by the formula:

$$\langle \mathbf{vc} \rangle (kT) = \frac{1}{T}\int_{(k-1)T}^{kT} \mathbf{vc}(t)\,dt = \sum_{r=0}^{r=P-1} \frac{tr}{T}\mathbf{vcr} = \sum_{r=0}^{r=P-1} \frac{tr}{T}\mathbf{OM\,r} = \mathbf{OM} \quad (1)$$

In this expression, t_r is the activation duration of the vector $\mathbf{v_{cr}}$.

Since $T = \sum_{r=0}^{r=P-1} tr$, we have $\sum_{r=0}^{r=P-1} \frac{tr}{T} = 1$. Consequently, the point M, define as $\mathbf{OM} = \langle \mathbf{v_c} \rangle (kT)$, can be considered as the barycenter of the P points M_r, with barycentric coordinates t_r/T.
Moreover, as t_r is positive, M is inside the polyhedron **B.** So, when an average control is adopted, it is the ***entire*** volume of the polyhedron **B** that characterizes the inverter and not only its vertex.

How to find the barycentric coordinates

Let us suppose that we have found a M point which allows to obtain the desired voltages for the load.
We know that M belongs to the polyhedron **B** and can be considered as the barycenter of P vertex of the polyhedron **B**. The problem is to find the barycentric coordinates. In fact, it is possible to find less than P vertex whose M is the barycenter.
The most practical interesting case consists in taking n+1 vertex which generate the vectorial space $\mathbf{F_n}$.
First, because it will be always possible to find such n+1 vertex of the polyhedron B whose M will be the barycenter. The reason is that the dimension of the vectorial space $\mathbf{F_n}$ associated to the n-leg VSI is

n. Second, in this case there will be a unique solution for the barycentric coordinates. For a three leg VSI for example, the common sequences use effectively only four vertex (Fig 4 and Fig 5).

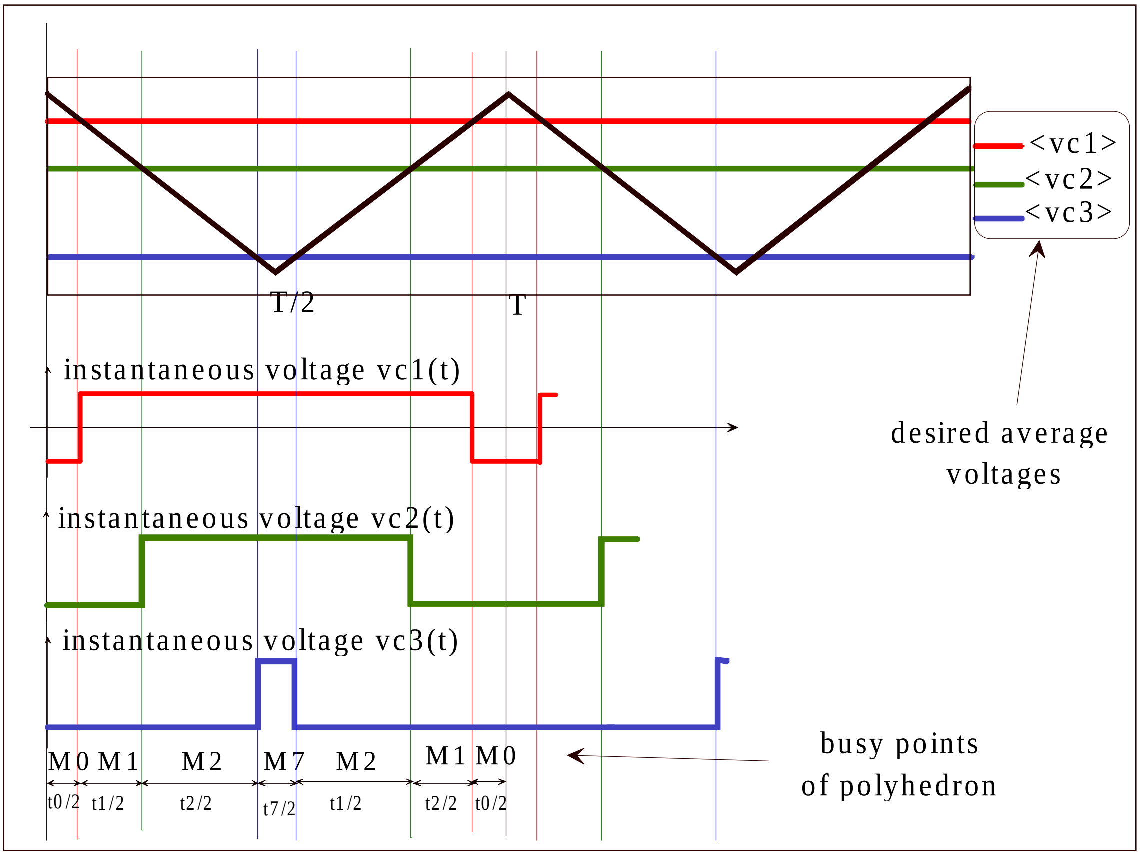

Fig 4: example of points (M_0, M_1, M_2, M_7) used in a symetric Intersective Pulse Width Modulation (Suboscillation method).

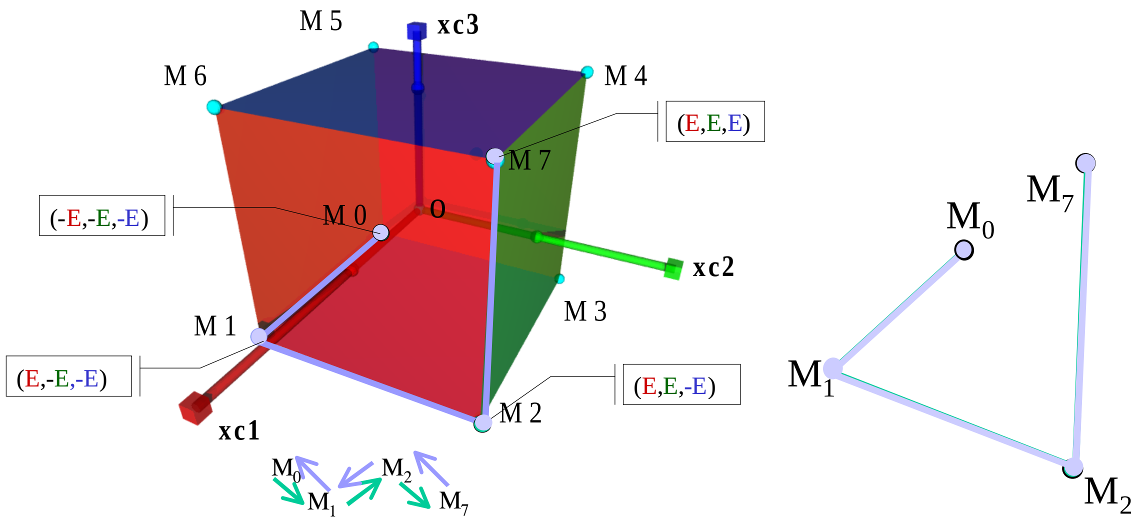

Fig 5: Representation of busy M_k points (M_0, M_1, M_2, M_7) in the sequence of symetric Intersective Pulse Width Modulation chosen in Fig 4.

The most common case: (n + 1) vertex and barycentric coordinates.

We consider a M point inside the polyhedron B. Then, we find n+1 vertex which create a $\mathbf{B_{n+1}}$ polyhedron whose M belongs. For the 3-leg inverter on Fig 4 we have chosen, to obtain the desired voltages, the points M_0, M_1, M_2 and M_7. We can find these n+1 vertex for example by taking, by successive tests of euclidian distances, the n+1 nearest vertex of M which generate the vectorial space $\mathbf{F_n}$. Let us note these n+1 vertex N_k, $k \in \{1,...,n+1\}$. So we can adapt the formula (1) to our particular case to obtain:

$$\mathbf{OM} = \sum_{k=1}^{k=n+1} \frac{t_k}{T} \mathbf{ON_k} \qquad (2)$$

As $T = \sum_{k=1}^{k=n+1} t_k$, we have $\sum_{k=2}^{k=n+1} \frac{t_k}{T} \mathbf{ON_1} = \mathbf{ON_1}$. The expression (2) is then equivalent to:

$$\mathbf{N_1M} = \sum_{k=2}^{k=n+1} \frac{t_k}{T} \mathbf{N_1N_k} \;. \qquad (3)$$

On the basis of this formula we will find, by use of classic vectorial properties, the barycentric coordinates t_k/T.

1. A free family

As the n+1 points N_k generate the vectorial space $\mathbf{F_n}$, the family of n vectors $\{\mathbf{N_1N_k}, 1<k<n+2\}$ is free. Consequently the determinant, $\det(\mathbf{N_1N_2}, \mathbf{N_1N_3}, ..., \mathbf{N_1N_{n+1}})$, is different from zero:

$$\det(\mathbf{N_1N_2}, \mathbf{N_1N_3}, ..., \mathbf{N_1N_{n+1}}) \neq 0 \qquad (4)$$

2. Mixed product

In a n-dimensional vectorial space $\mathbf{F_n}$, the mixed product of n vectors $\mathbf{w_k}$ is simply the determinant of a matrix, elaborated by concatenation of the n vectors $\mathbf{w_k}$:

$$(\mathbf{w_1}|\mathbf{w_2}|..|\mathbf{w_n}) = \det(\mathbf{w_1}, \mathbf{w_2}, ..., \mathbf{w_n}) \;.$$

The following property will be used : if exist k and r, $k \neq r$, as $w_k = w_r$ than $(\mathbf{w_1}|\mathbf{w_2}|..|\mathbf{w_n}) = 0$ (5)

3. Barycentric coordinates

$\forall k, k \in \{2,...,n+1\}$, we can express t_k with the n vectors $\mathbf{N_1N_j}$:

$$\boxed{t_k = T \frac{(\mathbf{N_1N_2} \,|\, \mathbf{N_1N_3} \,|\, ... \,|\, \mathbf{N_1N_{k-1}} \,|\, \mathbf{N_1M} \;\, |\, \mathbf{N_1N_{k+1}} \,|\, ... \,|\, \mathbf{N_1N_{n+1}})}{(\mathbf{N_1N_2} \,|\, \mathbf{N_1N_3} \,|\, ... \,|\, \mathbf{N_1N_{n+1}})}} \qquad (6)$$

Then t_1 is found by the following expression: $t_1 = T - t_2 - t_3 - \ldots - t_{n+1}$.

Proof:

To obtain this formula we have only to consider each term of the equation (3) and apply to them the following application ϕ defined by: $\phi(\mathbf{w}) = (\mathbf{N_1N_2}|\mathbf{N_1N_3}|...|\mathbf{N_1N_{k-1}}| \;\mathbf{w}\; |\mathbf{N_1N_{k+1}}|...|\mathbf{N_1N_{n+1}})$.

The property (5) implies the nullity of n terms of the equation (3) since:

$$(\mathbf{N_1N_2}|\mathbf{N_1N_3}|...|\mathbf{N_1N_{k-1}}| \;\mathbf{N_1N_r}\; |\mathbf{N_1N_{k+1}}|...|\mathbf{N_1N_{n+1}}) = 0 \;\text{ if } r \neq k$$

The equation (3) becomes then:

$$(\mathbf{N_1N_2}|\mathbf{N_1N_3}|...|\mathbf{N_1N_{n+1}}) \;\frac{t_k}{T} = (\mathbf{N_1N_2}|\mathbf{N_1N_3}|...|\mathbf{N_1N_{k-1}}|\mathbf{N_1M} \;\, |\mathbf{N_1N_{k+1}}|...|\mathbf{N_1N_{n+1}})$$

The property (4) allows us to divide by $(\mathbf{N_1N_2}|\mathbf{N_1N_3}|...|\mathbf{N_1N_{n+1}})$ to give the announced result.

4. Analysis of results

The expression of t_k is easy to implement because, for each t_k, there is only one vector, $\mathbf{N_1M}$, whose n coordinates change. The others vectors $\mathbf{N_1N_k}$ are constant and have to be calculated only one time.

The relation (6) shows that to obtain t_k we have, in general, to solve an algebraic system of n equations with n unknowns:

$$\mathbf{(t)} = A\,\mathbf{(<v_c>)} + \mathbf{(b)},$$

with **(b)** a constant vector, $(\mathbf{t}) = (t_2 \;\; t_3 \;\; \ldots \;\; t_{n+1})^t$ and $(<\mathbf{v_c}>) = (<v_{c1}> \;\; <v_{c2}> \;\; \ldots \;\; <v_{cn}>)^t$. The matrix A characterizes the polyhedron B_{n+1}.

Taking account the properties of the polyhedron B_{n+1} can lead to reductions. For example, in a few particular cases the matrix A is triangular. Then it is possible to obtain simply the solution t_k because A^{-1}, the inverse matrix of A, is also triangular. We explicit this last case for a 3-leg inverter in a next paragraph. An explicit research of the duration t_k is then no more useful because they can be considered as the intersection between a triangular shape wave and 3 horizontal lines (we find out the Suboscillation Method which is one way to implement the results).

Other cases

1. More than n+1 vertex

When the number k of vertex is higher than n+1, the decomposition of **OM** is not single. This will lead to more commutations and is not used for this reason.

2. Less than n+1 vertex

When the number k of vertex is less than n+1, the barycentric resolution is not always possible. It depends on the position of M in the polyhedron B. M must belong to a polyhedron generated by the k vertex considered. For example, for a 3-leg inverter M must be inside a triangle defined by three vertex, or inside the line segment defined by two vertex. These cases are used to optimize the number of commutations [2],[12].

Study of a 3-level VSI

We consider a carrier-based PWM with a period carrier T.

Comparison with Suboscillation Method

Let us prove that the Suboscillation Method is one way to determine the instants of commutation without calculate them explicitly.

We suppose that the desired average voltages at kT instant are those represented on Fig 4. The busy points of the polyhedron are then M_0, M_1, M_2, M_7. We have also defined a corresponding point M by $\mathbf{OM}=\langle \mathbf{v_c}\rangle(kT)=\langle v_{c1}\rangle\,\mathbf{x_{c1}}+\langle v_{c2}\rangle\,\mathbf{x_{c2}}+\langle v_{c3}\rangle\,\mathbf{x_{c3}}$.

Let us identify (N_1, N_2, N_3, N_4) to (M_7, M_0, M_1, M_2). Then, our method gives us the durations t_0, t_1, t_2 corresponding to M_0, M_1 and M_2 vertex:

$$t_0=T\frac{\left(\overrightarrow{M_7M}\,\middle|\,\overrightarrow{M_7M_1}\,\middle|\,\overrightarrow{M_7M_2}\right)}{\left(\overrightarrow{M_7M_0}\,\middle|\,\overrightarrow{M_7M_1}\,\middle|\,\overrightarrow{M_7M_2}\right)};\; t_1=T\frac{\left(\overrightarrow{M_7M_0}\,\middle|\,\overrightarrow{M_7M}\,\middle|\,\overrightarrow{M_7M_2}\right)}{\left(\overrightarrow{M_7M_0}\,\middle|\,\overrightarrow{M_7M_1}\,\middle|\,\overrightarrow{M_7M_2}\right)};\; t_2=T\frac{\left(\overrightarrow{M_7M_0}\,\middle|\,\overrightarrow{M_7M_1}\,\middle|\,\overrightarrow{M_7M}\right)}{\left(\overrightarrow{M_7M_0}\,\middle|\,\overrightarrow{M_7M_1}\,\middle|\,\overrightarrow{M_7M_2}\right)};$$

$$t_7=T-t_0-t_1-t_2$$

The development of these expressions gives:

$$t0=T\frac{4E^2(\langle vc1\rangle-E)}{-8E^3}\;;\; t1=T\frac{4E^2(\langle vc2\rangle-\langle vc1\rangle)}{-8E^3}\;;\; t2=T\frac{4E^2(\langle vc3\rangle-\langle vc2\rangle)}{-8E^3}\;.$$

We can rewrite these expressions:

$$-4E\,\frac{t_0/2}{T}+E=v_{c1}\;;\;-4E\,\frac{(t_0+t_1)/2}{T}+E=v_{c2}\;;\;-4E\,\frac{(t_0+t_1+t_2)/2}{T}+E=v_{c3}\;.$$

We find out the intersections of three horizontal lines $\langle v_{c1}\rangle$, $\langle v_{c2}\rangle$ and $\langle v_{c3}\rangle$ with a triangular shape varying between – E and E : $E-4E\frac{t}{T}$ for $0 < t < T/2$ et $-3E+4E\frac{t}{T}$ for $T/2 < t < T$.

Comparison with SVM Method

For a symetric three phase load which is star connected with neutral isolated (Fig 6) we have the classic relations between voltages:

- $u_{c1} = v_{c1} - v_{cN} = \frac{1}{3}(2\,v_{c1} - v_{c2} - v_{c3})$;
- $u_{c2} = v_{c2} - v_{cN} = \frac{1}{3}(-v_{c1} + 2\,v_{c2} - v_{c3})$;
- $u_{c3} = v_{c3} - v_{cN} = \frac{1}{3}(-v_{c1} - v_{c2} + 2\,v_{c3})$;

The image of the vector $\mathbf{v_c} = v_{c1}\,\mathbf{x_{c1}} + v_{c2}\,\mathbf{x_{c2}} + v_{c3}\,\mathbf{x_{c3}}$ is then the vector $\mathbf{u_c} = u_{c1}\,\mathbf{x_{c1}} + u_{c2}\,\mathbf{x_{c2}} + u_{c3}\,\mathbf{x_{c3}}$.
As $u_{c1} + u_{c2} + u_{c3} = 0$, this vector belongs always to a plane which can be considered as a complex plane.
If we consider $\mathbf{u_c}$ as a complex vector $\underline{\mathbf{uc}}$, we find the following expression:

$$\underline{\mathbf{uc}} = \sqrt{\frac{2}{3}}\left[+vc1\,1 + vc2\,a + vc3\,a^2\right] \text{ with } a = e^{j\frac{2\pi}{3}} .$$

This vector is effectively proportionnal to the classic phasor $\frac{2}{3}\left[+vc1\,1 + vc2\,a + vc3\,a^2\right]$.

It is possible to characterize the inverter for this kind of load by the image of the cube. We find that the image of the vextex of the cube are the vertex M_{kp} of the well known hexagon in complex plane represented Fig 7.
Consequently, the SVM formulation is effectively a particular case of our formulation.

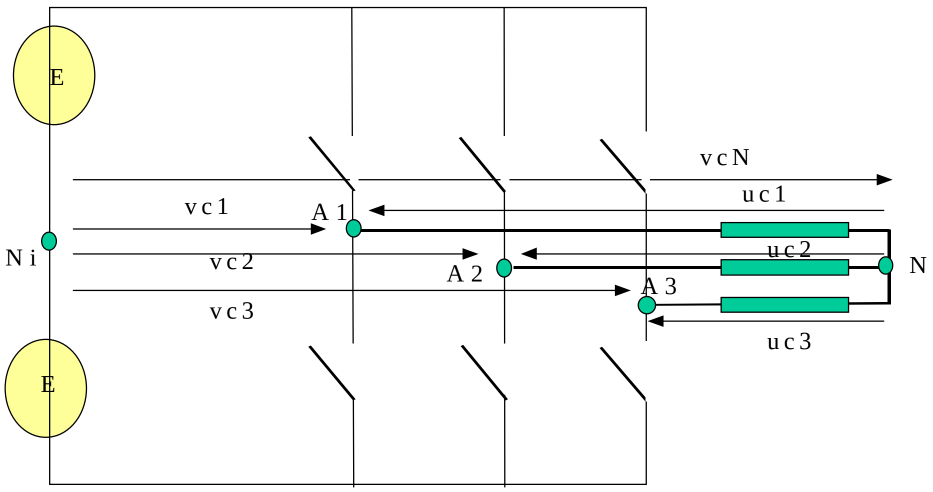

Fig 6: Representation of a 3-leg 2-level VSI

Conclusion

We prove that, even with a characterization of VSI in a n-dimensional vector space F_n, it is possible to calculate explicitly, as the SVM allows it in the complex plane, the conduction durations of the VSI switches. The method that we propose relies on a geometrical vectorial VSI characterization which enables, as the SVM, optimization, because of using geometrical tools as Euclidian distance, barycenter and scalar or vectorial products. But, contrary to the SVM, the desired vector $\mathbf{v_c}$ does not have to belong to a 2-dimensionnal vectorial space. We have explicited a few aspects of this approach in the well kwown case of 3-leg 2-level VSI. So we have showed the relations with the SVM method and with the Suboscillation method. The vectorial feature of the method allows to use it also for multilevel VSI or instaneous control as the Direct Torque Control of electrical machines.

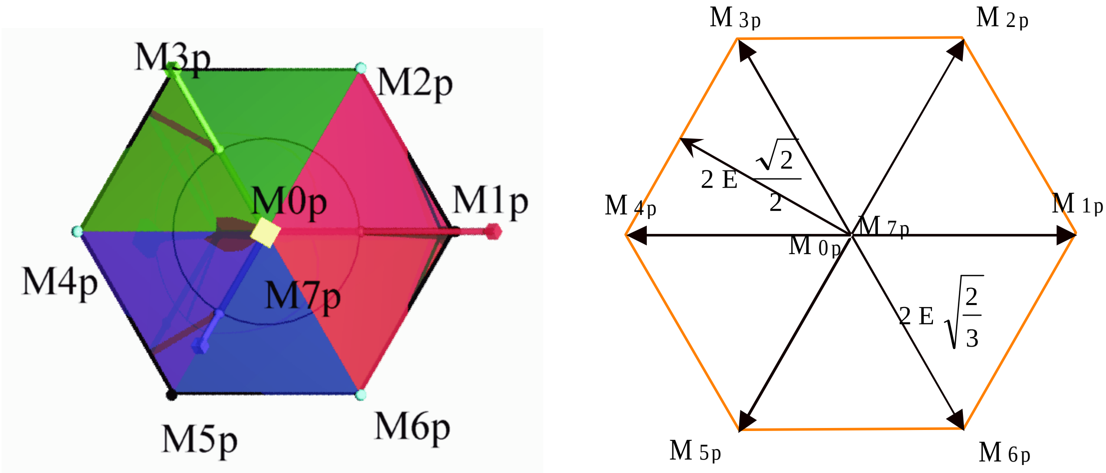

Fig 7: Images of the cube for a three-phase load which is star connected without neutral isolated.

References

[1] **Ali S., Kazmierkowski** " Current regulation of four-leg PWM-VSI" , IECON conference, Vol. 3/4, Aachen (Germany), August 1998, pp1853-1858

[2] **Blasko V.** " Analysis of a hybrid PWM based on modified space vector and triangle-comparison methods" , IEEE Transactions on industry applications, Vol. 33, n°3, May/june 1997

[3] **Cacciato M., Consoli A., Scarcella G., Testa A.** " A digital current control reducing common mode emissions " Electrimacs'99 Lisboa

[4] **Chung Dae-Woong, Kim J-S, Sul S-K** " Unified Voltage Modulation technique for real Time Three-Phase Power Conversion " IEEE Transactions on Industry applications, Vol. 34, n°2, Mars/April 1998

[5] **Dastfan A., Gosbell V.J., Don Platt** "Control of a New Active Power Filter Using 3-D Vector Control" IEEE Trans. on Power Electron. vol 15 n°1, january 2000 p5-12

[6] **Edelmoser K.H., Erhartt L.L.**, "Common Mode Disturbances in Force Controlled Rectifier Systems", PCIM'99 Power Conversion June 1999 (Nrnberg, Germany) proceedings p547-552

[7] **Erdman J.M., Kerkman R.J., Schlegel D.W., Skibinski G.** " Effect of PWM Inverters on AC Motor Bearing Currents and Shaft Voltages ", IEEE Transactions on Industry Applications, vol 32 n°2, pp 250-259, Mar/April 1996

[8] **François B., Bouscayrol A.**, "Design and modelling of a five-phase voltage-source inverter for two induction motors", EPE Conference Lausanne, september 1999.

[9] **Holtz J.** "Pulsewidth Modulation – A Survey" - IEEE Transactions on Industrial Electronics vol 39 n°5, december 1992

[10] **Julian A., Oriti G., Lipo T.** "Elimination of Common Mode Voltage in three phase sinusoidal Power Converters" IEEE Trans. on Power Electron. vol 14 n°5, sep1999 p982-98

[11] **Ogasawara S., Akagi H** " Circuit Configurations and Performance of the Active Common-Noise Canceler for reduction of Common-Mode Voltage Generated by Voltage-Source PWM Inverters " IAS 2000 Conference, september 2000, Roma

[12] **Semail E.**, "Tools and studying method of polyphase electrical systems. Generalisation of the space vector theory." Thesis, University of Lille I, june 2000, France (text in French)

[13] **Toliyat H.A., Waikar S. P., Lipo T.A.**, " Analysis and Simulation of Five-Phase Synchronous Reluctance Machines Including Third Harmonic of Airgap MMF", IEEE Transactions on Industry Applications, vol 3 n°2, pp 332-339, 1998

[14] **Zhao Y., Lipo A.** " Space Vector PWM Control of Dual Three-Phase Induction Machine Using Space Vector Decomposition" IEEE Trans. on Ind. Applicat., vol 31 n°5, Sep/oct 1995.