



Science Arts & Métiers (SAM)

is an open access repository that collects the work of Arts et Métiers Institute of Technology researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: <https://sam.ensam.eu>
Handle ID: <http://hdl.handle.net/10985/23861>



This document is available under CC BY-NC-ND license

To cite this version :

Amirhossein KHEZRI, Lazhar HOMRI, Alain ETIENNE, Jean-Yves DANTAN - An integrated resource allocation and tolerance allocation optimization: A statistical-based dimensional tolerancing - Procedia CIRP - Vol. 114, p.88-93 - 2022

Any correspondence concerning this service should be sent to the repository

Administrator : scienceouverte@ensam.eu



17th CIRP Conference of Computer Aided Tolerancing

An integrated resource allocation and tolerance allocation optimization: A statistical-based dimensional tolerancing

Amirhossein Khezri^{1*}, Lazhar Homri¹, Alain Etienne¹, and Jean-Yves Dantan¹

¹ LCFC, Arts et Métiers–ParisTech, Université de Lorraine, F-57000 Metz, France
{amir_hossein.khezri, lazhar.homri, alain.etienne, jean-yves.dantan}@ensam.eu

* Corresponding author. *E-mail address*: amir_hossein.khezri@ensam.eu

Abstract

Today's industrial world is facing rising demand for highly reliable and safe products. Complex industries, such as automobiles, medical, and aircraft, require a well-designed engineering plan which has a comprehensive understanding of the various certainties and uncertainties that occur in reality. Consequently, the need for reliable and precise parts has impacted the tolerancing activity. Key functions of complex products can often be realized by high precision part use. Thus, producers are confronted with high-quality requirements, cost pressure, and a rising number of demands. The introduction of new technologies and the need to meet the requirements have broadened the scope of tolerancing. In this paper, a statistical tolerance allocation model is developed to study the economic impact of allocated tolerances on an assembled product. The problem is aimed at optimizing the allocated tolerances to each part of the product while minimizing manufacturing costs. A modular cost model is proposed to determine the manufacturing costs related to each activity and part. The manufacturing costs include processing cost, inspection cost, scrap cost, assembly cost, and warranty cost. Furthermore, a genetic algorithm is adapted to study the applicability of the model developed on an exemplary assembled product.

© 2022 The Authors. Published by Elsevier B.V.

This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0>)
Peer-review under responsibility of the scientific committee of the 17th CIRP Conference on Computer Aided Tolerancing

Keywords: Resource allocation; Tolerance allocation; Statistical tolerancing; Optimization.

1. Introduction

Today's industrial world is facing rising demand for highly reliable and safe products which have a wide range of applications in complex industries, such as automobiles, medical, and aircraft. These industries require a well-designed engineering plan which has a comprehensive understanding of the various certainties and uncertainties that occur in reality. At all stages of product development and throughout the product life cycle, uncertainty is ubiquitous and incurs risk. The risk can impact the product performance(s), process scheduling, market acceptance, or the whole business. To mitigate these risks and to reduce their effects, many engineering design activities are

performed to look into thoroughly the concepts of uncertainty, risk, and tolerances [1].

On these bases, the introduction of new technologies has broadened the scope of tolerancing. The performance of complex products ultimately hinges on their manufacturing and assembly to current internal and external circumstances. Key functions of complex products can often be realized with high precision part use. Thus, producers face high-quality requirements, cost pressure, and a rising number of product variants. To determine the effects of tolerance and to understand the contributions of tolerances on the system behavior, it is necessary to identify the relationships between tolerances and functional characteristics through a set of experiments or numerical simulations.

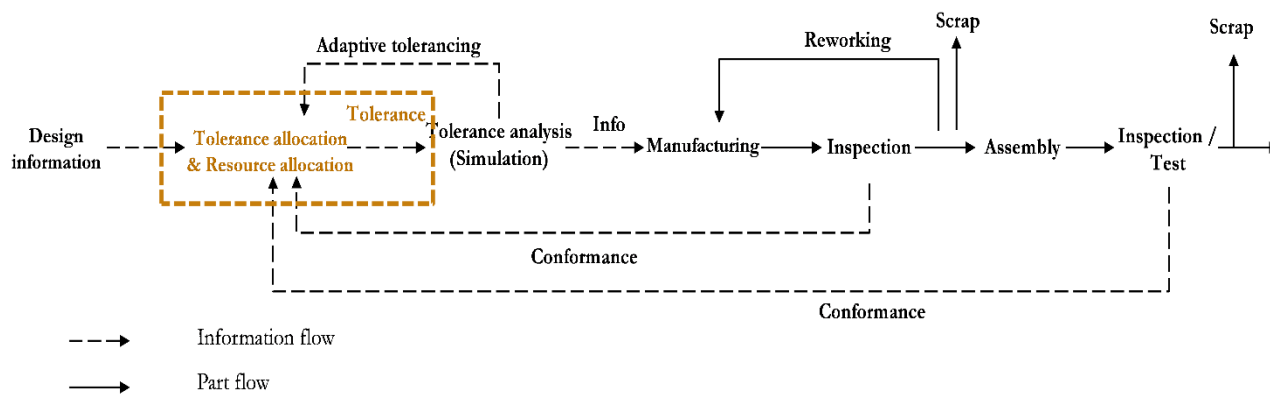


Fig. 1. Identification of tolerance’s role in an assembled product life

Tolerancing context can be classified into two distinct categories as tolerance analysis and tolerance allocation. Tolerance analysis is a method to verify the functionality of a design after tolerances have been specified on each part. On contrary, tolerance allocation, which is also called tolerance synthesis, involves the assignment and the distribution of the values of adequate tolerances [2,3].

model and support a mathematical model. In Section 4, an overrunning clutch mechanism is illustrated and the results of the proposed model are analyzed. Finally, Section 5 summarizes this paper and provides an outlook on prospects.

2. State of the art

In this section, we study relevant publications in the field of tolerance allocation and resource allocation to provide a better understating of these problems. Tolerance allocation involves the assignment and the distribution of the values of adequate tolerances [2,3]. González & Sánchez [4] proposed an innovative methodology for statistical tolerance allocation with dependent variables. The dependence structure is estimated from the manufacturing process where the multivariate process variability is the consequence of a set of independent factors. Cheng & Tsai. [5] developed a closed-form solution to the statistical tolerance allocation problem using the Lambert W function to solve the problem. Sanz-Lobera et al. [6] extended a parametric cost-tolerance function deriving the parameters from each manufacturing process based on statistical distributions of manufactured parts.

Furthermore, Liu et al. [7] employed various types of manufacturing cost functions (i.e. exponential, reciprocal power, and polynomial) to choose the best one with the minimum fitting error. Then a closed-form method for statistical tolerance allocation was proposed to fulfil expected quality with less manufacturing cost. Khodaygan [8] studied an asymmetric tolerance allocation problem providing a high quality and cost-effective product. Wang et al. [9] developed a Bayesian-based method for a statistical tolerance allocation considering the model parameter uncertainty and the change of design factor within the limited tolerances. The review of the relevant studies in parametric cost models and statistical tolerance allocation problem illustrates how tight is the link between resource selection and tolerance allocation due to the process deviation associated with each resource.

In this regard, several studies have been carried out with a focus on process and resource selection. Mustajib [10] integrated tolerance allocation in an interchangeable assembly to simultaneously evolve suitable combinations of a manufacturing facility. The model results a cost-effective assembled product with less clearance variation. Yeo et al. [11]

Nomenclature

N	Set of parts
O	Set of resources
LSL_i	Lower specification limit for part i
USL_i	Upper specification limit for part i
X_i	Geometrical deviation on part i
Y	Functional requirement
μ_i	The nominal value of dimension for part i
$\sigma_{i,j}$	Process deviation of operation j for part i
t_i	Allocated tolerance to part i
$a_{i,j}$	1 if resource j is allocated to part i , 0 otherwise.
σ_y	Assembled product functional requirement standard deviation
t_y	Assembled product tolerance
γ_i	Part i conformity ratio
λ	Assembled product conformity ratio
α	Type I failure rate
β	Type II failure rate

Therefore, this paper aims at developing strategies for an optimal tolerance allocation, assuring an economic production of a functional product through determining the role of tolerance in a product cycle life (Fig. 1). As it is highlighted in Fig. 1, this paper is divided into two sub-problems, the development of a mathematical model for tolerance optimization integrating decisions on allocating optimal tolerances and resources, accordingly.

This paper is structured as follows: section 2 provides a state of the art on tolerance allocation and resource allocation. Section 3.1 and Section 3.2 define the statistical tolerancing

developed a cost-tolerance model considering various manufacturing processes. The model uses an expert system approach to optimize the process sequences. Natarajan et al. [12] developed an integrated tolerance synthesis and process selection problem by considering process capability and costs of non-conformance. Jing et al. [13] developed a Monte Carlo-based simulation and self-adaptive differential evolution to take the impact of multiple alternative manufacturing processes on the part. Hallmann et al. [14] illustrated a sampling-based tolerance-cost optimization with a focus on machine/process selection. The model takes into account the machine characteristics of alternative machines, such as process capabilities and manufacturing. The problem is to find an optimal solution for the tolerance allocation problem with the integration of the machine selection problem using a genetic algorithm.

In summary, parametric cost modeling has permanently been in focus for decades. The fact that the model depends on parametric data which lacks in taking into account uncertainties has been neglected. Therefore, statistical and modular cost modeling is an alternative approach to estimate manufacturing cost where the model is eased of the direct impact of allocated tolerances. Consequently, in this paper, a statistical-based cost model is proposed, simultaneously considering the contribution of allocated tolerances and resources assigned to operate processes. The next section describes the main research problem and demonstrates the mathematical formulation of this problem.

3. Statistical-based cost model

In the following section, a statistical resource and tolerance allocation model is described and formulated to be optimized.

3.1. Description

In this section, a tolerancing problem is proposed, which assures an economic production of a functional product. Within this section, a statistical-based cost model is described. The statistical-based model is applied using a modular cost model which estimates product final cost involving several activities such as processing, inspection, scraping, etc. The development of the statistical model leads us to study the economic impact of allocated tolerance (t_i) and assigned resources ($a_{i,j}$) to an assembled product. The model takes into account five activities: processing, inspection, scraping, assembly, and warranty, respectively. The activities are weighted by the efficiency of the related activities which are correlated to the conformity ratio of the assembled product and its parts.

Moreover, the assembled product and the parts conformity ratios depend on the allocated tolerances and associated resources. Consequently, to examine the assembled product cost, a statistical model is proposed where activities' costs are constant, however, activities' weights are correlated to the allocated tolerances and resources. Additionally, for sake of simplicity, the model follows several assumptions:

- (1) Tolerance allocation problem is defined on dimensional tolerancing of a designed part.

- (2) A generic form of conformity rate estimator is developed based on normal distribution.
- (3) Dimensions are independent, therefore, the sole dependency in this model is between parts tolerances and functional requirement.

The tolerance of a part can be defined as the permissible variation in measurements deriving from the nominal value. It can be expressed as follow:

$$t_i = USL_i - \mu_i = \mu_i - LSL_i = 3\sigma_{i,j} \quad \begin{array}{l} , \forall i \in N \\ , \forall j \in O \end{array} \quad (1)$$

where USL and LSL express upper and lower specification limits and μ denotes nominal value. However, parts' tolerances are dependent on process standard deviations (σ).

In this model, to allocate tolerances to the individual parts of an assembled product, manufacturing activities' costs are assessed. The model includes activities such as processing, inspection, scraping, assembly, and warranty. Furthermore, a mathematical model is proposed to illustrate the theoretical concept of the model and formulate the problem in a generic form.

3.2. Formulation

Developing a statistical model, allows us to estimate economic impacts and conformity of allocated tolerances to an assembled product. The conformity ratio of an individual part (γ) is calculated as below:

$$\gamma_i = P\left(\frac{\mu_i - t_i}{a_{i,j} \times \sigma_{i,j}} \leq X_i \leq \frac{\mu_i + t_i}{a_{i,j} \times \sigma_{i,j}}\right) \quad \begin{array}{l} , \forall i \in N \\ , \forall j \in O, a_{i,j} \neq 0 \end{array} \quad (2)$$

Furthermore, to calculate the assembled product conformity ratio (λ), the design function of the mechanism is required. A design function determines the characteristic value (for instance y) of the assembled product by the characteristic values of parts ($x_i | i \in N$), i.e., $y = f(x_1, x_2, \dots, x_n)$. Moreover, the assembled product variance (σ_y) can be approximated by the process deviation of the allocated resources and expresses as follow:

$$\sigma_y \approx f(\sigma_{1,1}, \sigma_{1,2}, \dots, \sigma_{2,1}, \sigma_{2,2}, \dots, \sigma_{i,j}) \quad \begin{array}{l} , \forall i \in N \\ , \forall j \in O \end{array} \quad (3)$$

Knowing Eq. (4), helps us to estimate the conformity ratio of the assembled product and it is equal to:

$$\lambda = P\left(\frac{\mu_y - t_y}{\sigma_y} \leq Y \leq \frac{\mu_y + t_y}{\sigma_y}\right) \quad (4)$$

On these bases, an activity-based cost objective including activities such as processing ($Cost_{Processing}$), inspection ($Cost_{Inspection}$), scraping ($Cost_{Scrap}$), assembly ($Cost_{Assembly}$), and warranty service ($Cost_{Warranty}$) is developed. In this problem, allocated tolerances and resources must satisfy the following objective:

$$Min Cost_{Total} = Cost_{Processing} + Cost_{Inspection} + Cost_{Scrap} + Cost_{Assembly} + Cost_{Warranty} \tag{5}$$

$$Cost_{Processing} = \sum_{i \in N} \sum_{j \in O} \frac{C_{m_{i,j}} \times a_{i,j}}{\gamma_i(1-\alpha) + (1-\gamma_i)\beta} \tag{6}$$

$$Cost_{Inspection} = \sum_{i \in N} \sum_{j \in O} \frac{C_{i,j} \times a_{i,j}}{\gamma_i(1-\alpha) + (1-\gamma_i)\beta} \tag{7}$$

$$Cost_{Scrap} = \sum_{i \in N} \sum_{j \in O} \frac{C_{s_i}(\gamma_i\alpha + (1-\gamma_i)(1-\beta)) \times a_{i,j}}{\gamma_i(1-\alpha) + (1-\gamma_i)\beta} \tag{8}$$

$$Cost_{Assembly} = \frac{C_{Asm}}{\lambda(1-\alpha) + (1-\lambda)\beta} \tag{9}$$

$$Cost_{Warranty} = \frac{C_{Warr}((1-\lambda)\beta)}{\lambda(1-\alpha) + (1-\lambda)\beta} \tag{10}$$

In this model, two common inspection failures are integrated, respectively Type I and Type II failure rates. Type I failure rate (α) happens once the process is confirmed however the inspection rejects it and Type II failure rate (β) occurs when a non-confirmed process returns as a confirmed process from the inspection. As a constraint to be met, the model has to take into account that each part can be processed with only one resource and this term is expressed in Eq. (11):

$$\sum_{j \in O} a_{i,j} = 1, \forall i \in N \tag{11}$$

4. An illustration and Results analysis

4.1. Overrunning clutch mechanism and manufacturing data

To validate the suggested model, a commonly used overrunning clutch mechanism (as shown in Fig. 2) is studied [15]. In this mechanism, the contact angle (Y) is the functional requirement and its value must be controlled within the range $6.99 \pm 1 \text{ deg}$. The function design depends on parts' geometrical deviations, i.e., hub (X_1), roller (X_2), and cage (X_3) and it is expressed in Eq. (12):

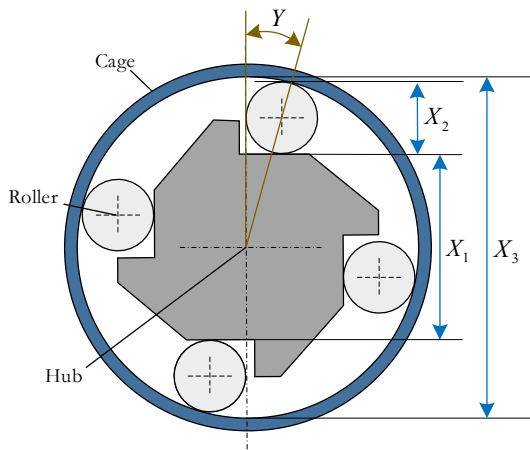


Fig.2. An overrunning clutch mechanism [15].

$$Y = f(X_1, X_2, X_3) = \arccos\left(\frac{X_1 + X_2}{X_3 - X_2}\right) \tag{12}$$

The nominal value of the parts ($\mu_i, i = 1, 2, 3$) are 55.2973 mm, 22.86 mm, and 101.6 mm, respectively. In this study, a Root Square Sum (RSS) is used which is well-known as an optimistic method to evaluate functional requirement deviation and expresses as follow:

$$\sigma_y = \sqrt{\sum_{i \in N} \sum_{j \in O} \left(\left| \frac{\partial y}{\partial x_i} \right|_{\mu_i} \sigma_{i,j} \times a_{i,j} \right)^2} \tag{13}$$

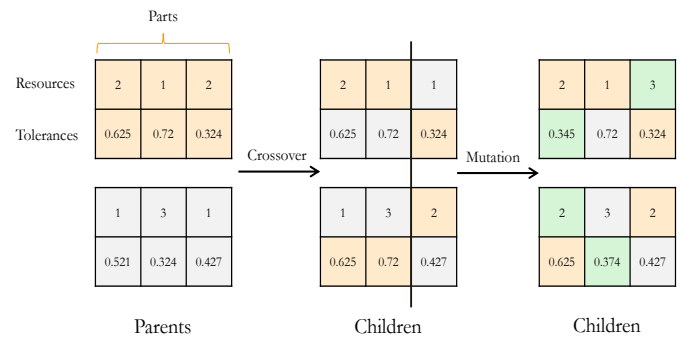


Fig. 3. Genetic algorithm chromosome, crossover, and mutation presentations.

For the sake of simplicity, the derivatives of Y in respect to $X_i (i = 1, 2, 3)$ are calculated and given: $\left| \frac{\partial Y}{\partial X_1} \right|_{\mu_{Hub}} = 0.1049$, $\left| \frac{\partial Y}{\partial X_2} \right|_{\mu_{Roller}} = 0.2084$, $\left| \frac{\partial Y}{\partial X_3} \right|_{\mu_{Cage}} = 0.1038$. Moreover, the manufacturing cost includes several activities such as processing, inspection, scrapping, and assembly. In Table (1), associated costs, process deviations, and inspection errors are provided to model the manufacturing cost.

Afterward, to analyze the system behavior and estimate the manufacturing cost, a genetic algorithm is developed using Python 3.7 with the following tuning parameters [16]: number of iterations = 1000, population size = 200, mutation probability = 0.04, crossover probability = 0.5, elite rate = 0.01. In Fig. 3, the chromosome representation and crossover and mutation operators are illustrated. A chromosome is structured of two sub genes. The first sub gene contains assigned resources' information to each part and the second sub gene includes allocated tolerances' information, accordingly. Afterward, crossover and mutation operators are used to generate new children out of parents. Moreover, the associated cost value of each child will be evaluated and the best child in each iteration of the generation will be selected. In the end, the best child among all dominated children will be selected which contains the optimal tolerances and resources information to be allocated to each part.

4.2. Results analysis

Following our last section and the application of overrunning clutch, the analysis is provided.

Table 1. Manufacturing data for the overrunning clutch mechanism

Parts	Hub			Roller			Cage		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
Resources	R1	R2	R3	R1	R2	R3	R1	R2	R3
Processing cost $Cm_{i,j}$ (cu)	2	3.15	3.5	3	2.5	2.95	2.95	3.15	4
Process deviation $\sigma_{i,j}$ (mm)	0.0566	0.0133	0.0100	0.0166	0.0300	0.0208	0.0208	0.0133	0.009
Inspection cost Ci_i (cu)	1			1.5			1		
Scrap cost Cs_i (cu)	0.5			0.5			0.5		
Product assembly cost C_{Asm} (cu)	3								
Inspection error α	0.0027								
Inspection error β	0.00005								
Note: cu = Cost unit									

The resulted optimal solutions for a variety of functional requirements are detailed in Fig. 4 and Fig. 5. Moreover, the trends in assembly conformity rate and manufacturing cost with change in tolerance of functional requirement are depicted in Fig. 4. As it has been illustrated in the figures, depending on the acceptable value for functional requirement, significant changes in resources, tolerances, conformities, and cost are realised. For instance, a tight value for tolerance of the functional requirement as $t_y = 0.07$ results 19.3 (cu) as the manufacturing cost of the assembled product, however, assembly conformity rate is equal to $\lambda_{0.07} = 89.33\%$.

On the other hand, if we loosen tolerance of the functional requirement to $t_y = 0.4$, the manufacturing cost decreases to 14.6 (cu) and would have an increase in assembly conformity rate to the value of $\lambda_{0.4} = 99.73\%$.

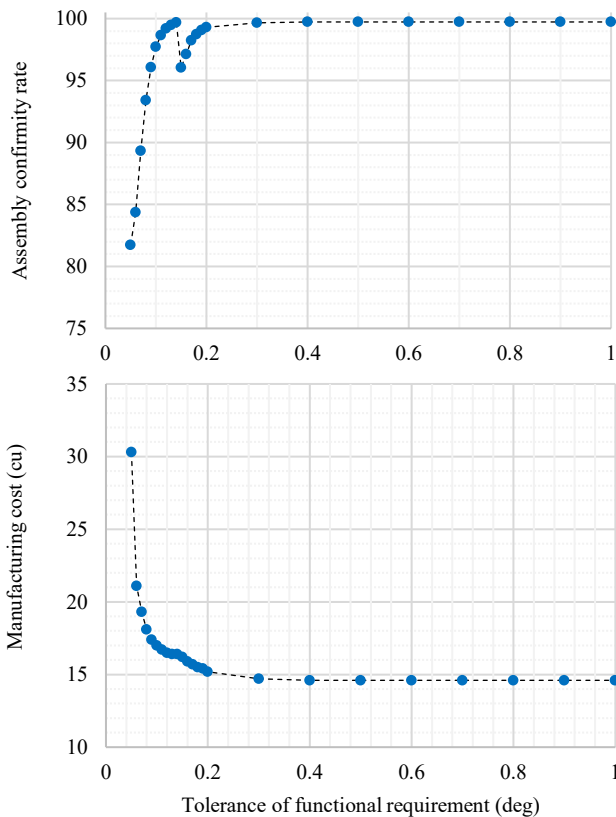


Fig. 4. Assembly conformity rate and manufacturing cost with respect to change in functional requirement

resources to operate the processes variate. Analysing allocated tolerances and resources to these two parts illustrates the impact of resources on the solutions. Correspondingly, the difference in allocated tolerances is significant. The reason is underlined with allocated resources and cost model structure. The model has more tendency toward allocating optimal resources with great impact rather than allocating tolerance with less impact.

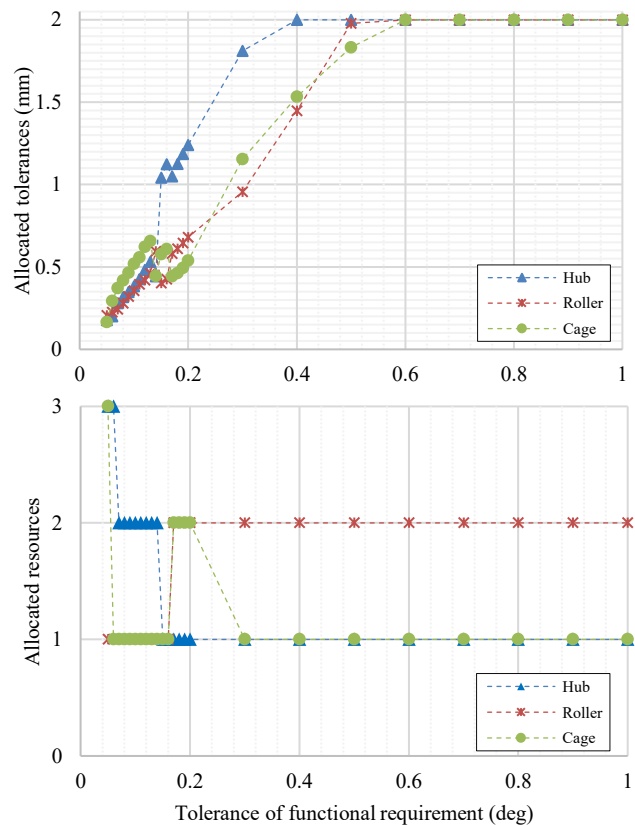


Fig. 5. Allocated tolerances and resources to process the parts concerning change in functional requirement

Additionally, studying obtained results in Fig. 5 expresses the best decisions on allocating optimal tolerances to parts and assigning the best resources to process the parts. As it can be read from Fig. 5, increasing the functional requirement impacts directly on allocated tolerances to the parts. However, the strength of impact is dependent on the available resources and the deduced derivation coefficients from the function design equation. For instance, in this product, derivation coefficients

Nevertheless, derivation coefficients are so close, available

for hub and cage are equal to $\left| \frac{\partial Y}{\partial X_1} \right|_{\mu_{Hub}} = 0.1049$ and $\left| \frac{\partial Y}{\partial X_3} \right|_{\mu_{Cage}} = 0.1038$, respectively. In accordance with Fig. 4, it can be deduced that the value of the functional requirement, defined by the customers as a requirement to be respected, can have significant impacts on the conformity rate of the assembled product, as well on manufacturing cost. It can be concluded that the decision on functional requirement has a direct relation with assembly conformity rate and indirect relation with manufacturing cost. The reason can be found in the allocated tolerances to each dimension and resources assigned to operate processes (given in Fig. 5). If the customer's decision is on a tight level of functional requirement, therefore, tighter tolerances would be allocated. Consequently, to satisfy the allocated tolerances on the parts, highly precise resources should be assigned to operate the process.

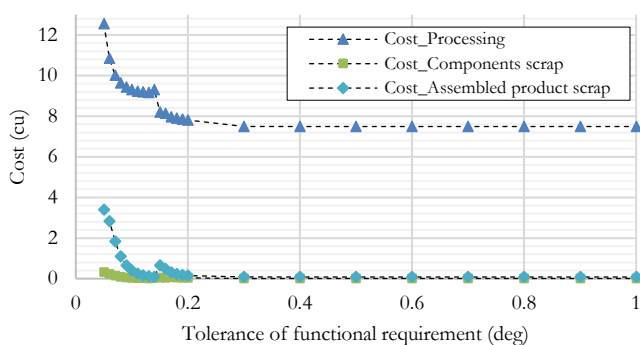


Fig 6. $Cost_{processing}$ and $Cost_{scrap}$ trends concerning change in functional requirement

Moreover, the costs associated to the activities are influenced by decision on allocated tolerances, assigned resources, and how tight is the functional requirement. For instance, the tendencies of $Cost_{processing}$ and $Cost_{scrap}$ over the taken decisions are depicted in Fig. 6.

5. Conclusion and Future Work

An assembled product is constructed of several parts, consequently, assembly conformity and correlated manufacturing cost depends on a variety of uncertainties and activities. Therefore, in this article, a statistical cost model was proposed to examine the uncertainties within two manufacturing problems: tolerance allocation and resource allocation. The model provides a modular cost assessment tool which is regardless of parametric cost models and can easily be adjusted depending on the current activities of the assembled production system. Finally, a genetic algorithm is used to allocate optimal tolerances and resources while minimizing the manufacturing cost. The application of the model on the overrunning clutch illustrated the tolerance and resource allocation consequences on the final manufacturing cost and the assembled product conformity rate.

In future work, the development of the proposed approach is to address selection strategies into the model and study the associated consequences on assembled product conformity and cost. Moreover, it is critical to enhance the statistical model where positional and rotational tolerances are also examined.

Acknowledgments

The authors gratefully acknowledge the support by AdeQuaT Project which is funded by the French Agence Nationale de la Recherche (ANR) and the Deutsche Forschungs-gemeinschaft (DFG) under the grant number ANR-19-CE10-0013-01.

References

- [1] Morse, E., Dantan, J.-Y., Anwer, N., Söderberg, R., Moroni, G., Qureshi, A., Jiang, X. & Mathieu, L. (2018). Tolerancing: Managing uncertainty from conceptual design. *Cheng, K.-M., & Tsai, J.-C. (2013). Optimal statistical tolerance allocation for reciprocal exponential cost-tolerance function. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 227(5), 650-656.*
- [2] Drake, P. J. (1999). *Dimensioning and tolerancing handbook*: McGraw-Hill New York.
- [3] Ji, S., Li, X., Ma, Y., & Cai, H. (2000). Optimal tolerance allocation based on fuzzy comprehensive evaluation and genetic algorithm. *The International Journal of Advanced Manufacturing Technology, 16(7), 461-468.*
- [4] González, I., & Sánchez, I. (2009). Statistical tolerance synthesis with correlated variables. *Mechanism and Machine Theory, 44(6), 1097-1107.*
- [5] Cheng, K.-M., & Tsai, J.-C. (2013). Optimal statistical tolerance allocation for reciprocal exponential cost-tolerance function. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 227(5), 650-656.*
- [6] Sanz-Lobera, A., Gómez, E., Pérez, J., & Sevilla, L. (2015). A proposal of cost-tolerance models directly collected from the manufacturing process. *International Journal of Production Research, 54(15), 4584-4598.*
- [7] Liu, S., Jin, Q., Dong, Y., & Wang, Y. (2017). A closed-form method for statistical tolerance allocation considering quality loss and different kinds of manufacturing cost functions. *The International Journal of Advanced Manufacturing Technology, 93(5-8), 2801-2811.*
- [8] Khodaygan, S. (2019). A multiple objective framework for optimal asymmetric tolerance synthesis of mechanical assemblies with degrading components. *The International Journal of Advanced Manufacturing Technology, 100(9-12), 2177-2205.*
- [9] Wang, J., Mao, T., & Tu, Y. (2020). Simultaneous multi-response optimisation for parameter and tolerance design using Bayesian modelling method. *International Journal of Production Research, 1-25.*
- [10] Mustajib, M. I. (2012). Concurrent engineering of tolerance synthesis and process selection for products with multiple quality characteristics considering process capability. *Makara Journal of Technology, 16(1), 2.*
- [11] Yeo, S., Ngoi, B., & Chen, H. (1996). A cost-tolerance model for process sequence optimisation. *The International Journal of Advanced Manufacturing Technology, 12(6), 423-431.*
- [12] Natarajan, J., Sivasankaran, R., & Kanagaraj, G. (2018). Bi-objective optimization for tolerance allocation in an interchangeable assembly under diverse manufacturing environment. *The International Journal of Advanced Manufacturing Technology, 95(5-8), 1571-1595.*
- [13] Jing, T., Tian, X., Liu, X., Hu, H., Zhang, M., & Li, B. (2020). A multiple alternative processes-based cost-tolerance optimal model for aircraft assembly. *The International Journal of Advanced Manufacturing Technology, 107(1), 667-677.*
- [14] Hallmann, M., Schleich, B., & Wartzack, S. (2021). Process and machine selection in sampling-based tolerance-cost optimisation for dimensional tolerancing. *International Journal of Production Research, 1-16.*
- [15] Feng, C.-X., & Kusiak, A. (1997). Robust tolerance design with the integer programming approach.
- [16] Singh, P., Jain, S., & Jain, P. (2004). A genetic algorithm based solution to optimum tolerance synthesis of mechanical assemblies with alternate manufacturing processes—benchmarking with the exhaustive search method using the Lagrange multiplier. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 218(7), 765-778*