# Technique de saut de cycles appliquée à l'homogénisation en régime viscoélastique-viscoplastique avec endommagement en vue de la prédiction de la réponse cyclique de composite PA66/GF

# *Cycle jump technique combined with mean-field micromechanics towards predicting the cyclic response of PA66/GF composites under viscoelastic-viscoplastic regime and damage mechanisms*

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## Résumé

Ce travail propose un cadre micromécanique avec endommagement probabiliste pour prédire la réponse contrainte-déformation uniaxiale et cyclique et l'endommagement progressif dans les composites polyamides renforcés de verre aléatoires. Motivé par différents modes de dégradation microscopiques observés expérimentalement, le mécanisme d'endommagement au voisinage des fibres est caractérisé par l'apparition et la coalescence de vides, dont l'évolution peut être formulée par une fonction de densité probabiliste de Weibull. En revanche, la dégradation progressive ductile de la rigidité initiale de la matrice est analysée via la théorie de l'endommagement continu. À cette fin, une méthode Mori-Tanaka (MT) 2N+1-phase combinée à l'approche d'analyse de champ de transformation (TFA) est établie dans un cadre unifié. De plus, la réponse viscoélastique et viscoplastique de la matrice polymère est formulée à l'aide d'un modèle phénoménologique composé de quatre branches de Kelvin-Voigt et d'une branche viscoplastique dans le cadre de la thermodynamique. La comparaison des prédictions numériques avec les données expérimentales démontre les capacités du modèle. Dans une seconde étape de ce travail, le schéma micromécanique est combiné avec la technique du saut de cycle afin de simuler des essais de fatigue cyclique modérée et élevée. Cette stratégie de modélisation est validée par comparaison avec des résultats expérimentaux.

## Abstract

This work proposes a probabilistic micromechanics damage framework to predict the uniaxial and cyclic stress-strain response and progressive damage in random glass-reinforced polyamide composites. Motivated by different microscopic degradation modes observed experimentally, the damage mechanism in the vicinity of the fibers is characterized by the onset and the coalescence of voids, whose evolution can be formulated through a Weibull probabilistic density function. In contrast, the ductile progressive degradation of matrix initial stiffness is analyzed via the continuum damage theory. Towards this end, a 2N+1-phase Mori-Tanaka (MT) method combined with the transformation field analysis approach (TFA) is established within a unified framework. Moreover, the rate-dependent viscoelastic and viscoplastic response of the polymer matrix phase is formulated through a phenomenological model consisting of four Kelvin-Voigt branches and a viscoplastic branch under the thermodynamics framework. Comparison of numerical predictions with experimental data demonstrates the model's capabilities. In a second step of this work, the micromechanics scheme is combined with the cycle-jump technique in order to simulate moderate and high cycle fatigue tests. This modeling strategy is validated through comparison with experimental results.

**Mots Clés :** composites thermoplastiques, endommagement, Mori-Tanaka, Analyse de champ de transformation, technique de saut de cycle
**Keywords :** thermoplastic composites, damage, Mori-Tanaka, Transformation Field Analysis, cycle jump technique

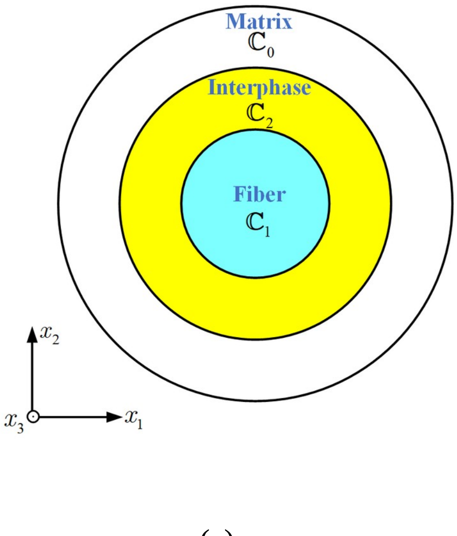
## 1. Introduction

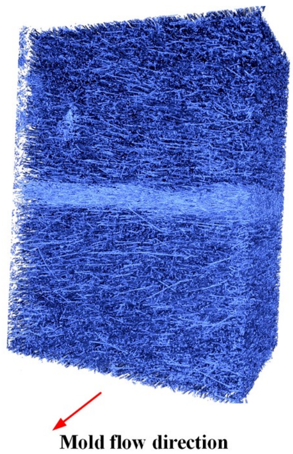
It is well recognized that the semi-crystalline polymer matrix composites exhibit substantial rate-dependent inelastic stress-strain behavior. Accordingly, this nonlinear response has been simulated by viscoelastic and viscoplastic (VE-VP) constitutive models. The inelastic behavior is typically more pronounced in the presence of microcrack, which significantly affects the global stress-strain response and hence the load-bearing capability of the polymer-based composites under various loading conditions.

Studies on composite materials can be performed through homogenization theories. The latter are good predictive tools for the composite's macroscopic response using as information the response of the individual phases and the microstructures. Micromechanics techniques, in general, can be categorized into two broad categories, namely the full-field and mean-field homogenization techniques. The former are more accurate but require computationally expensive numerical approaches, while the latter are analytical or semi-analytical but under strongly nonlinear mechanisms they provide moderate accuracy.

This work focuses on two aspects: a) combine the Mori-Tanaka (mean-field homogenization) scheme with the Transformation Field Analysis (TFA) strategy, including a specially designed coating layer between the matrix and the reinforcement that takes into account increased viscoplastic and damage effects at the interface [1,2], and b) implement a cycle jump procedure into the extended Mori-Tanaka/TFA micromechanics method and employing the extended theory to predict the high-cycle response of PA66/GF composites under various loading conditions. The developed framework considers the viscoelastic-viscoplastic and damage mechanisms for a number of cycles to establish the global evolution functions of all the internal variables (SDVs). Extrapolating the SDVs, such as strain, stress, damage variables, viscoelastic and viscoplastic strains over significant cycle lengths enables efficient implementation of the cycle jump procedure as proposed by Cojocaru and Karlsson [3]. This method permits an automatic jump length and extrapolating error control over the cycle jump period [4].

## 2. Mori-Tanaka/TFA micromechanics strategy with interphase layer




(a) (b)

*Fig. 1. (a) A three-phase Mori-Tanaka model with progressive interphase and void damage. (b) Skin-shell-core microstructure observed in the through-thickness of PA66/GF composites.*

The aim of the present work is to capture the rate-dependent stress-strain behavior of short glass fiber reinforced polyamide composites under both monotonic and oligocyclic loading by considering physically justified deformation and damage mechanisms, including viscoelasticity, viscoplasticity, interphase decohesion, and matrix ductile damage, as well as integrating the actual injection-induced fiber orientation distributions.

For the study of the PA66/GF short glass fiber composites, an extended Mori-Tanaka approach, combined with the transformation field analysis [5] has been developed, considering an interphase layer (Fig. 1a) between the matrix and the reinforcement. This addition permits to modify properly the mean-field strategy and provides more accurate results, closer to the predictions of full-field techniques [6,1].

The PA66 matrix is modeled as a viscoelastic-viscoplastic material with ductile damage. The glass fibers are assumed elastic media, while the interphase layer is considered of a similar nature with the matrix, having two main differences: i) while it has the same elastic properties with the PA66, the inelastic strains developed on this layer is directly linked to the matrix strain through a correction tensor Y [6]. ii) The damage developed at the interphase is more severe and is described by a discrete model using a Weibull probabilistic density function [1].

For a three-phase composite consisting of the fiber (subscript 1), the interphase (subscript 2), and the matrix (subscript 0) phases, the average strain $\varepsilon_r$ of the $r_{th}$ phase reads:

$$\varepsilon_r = T_r : \varepsilon_0 + T^p_{r0} : \varepsilon^p_0 + T^p_{r1} : \varepsilon^p_1 + T^p_{r2} : \varepsilon^p_2, \qquad \text{(Eq. 1)}$$

where r=1,2. $\varepsilon_0$ denotes average matrix strain and $\varepsilon^p_0, \varepsilon^p_1, \varepsilon^p_2$ represent the inelastic strains of the three phases. $T_r, T^p_{r0}, T^p_{r1}, T^p_{r2}$ are the fourth-order elastic and inelastic interaction tensors [6].

To account for multiple coated fiber orientations in a matrix, the above expression must be further extended. For an N-orientation coated fiber composite, the strains in the matrix, the ith fiber and the ith coating are written as follows [2]:

$$\varepsilon_0 = A_0 : \bar{\varepsilon} + A_{00} : \varepsilon^p_0 + \sum_{i=1}^{N} [A^i_{01} : \varepsilon^{p,i}_1 + A^i_{02} : \varepsilon^{p,i}_2], \qquad \text{(Eq. 2)}$$

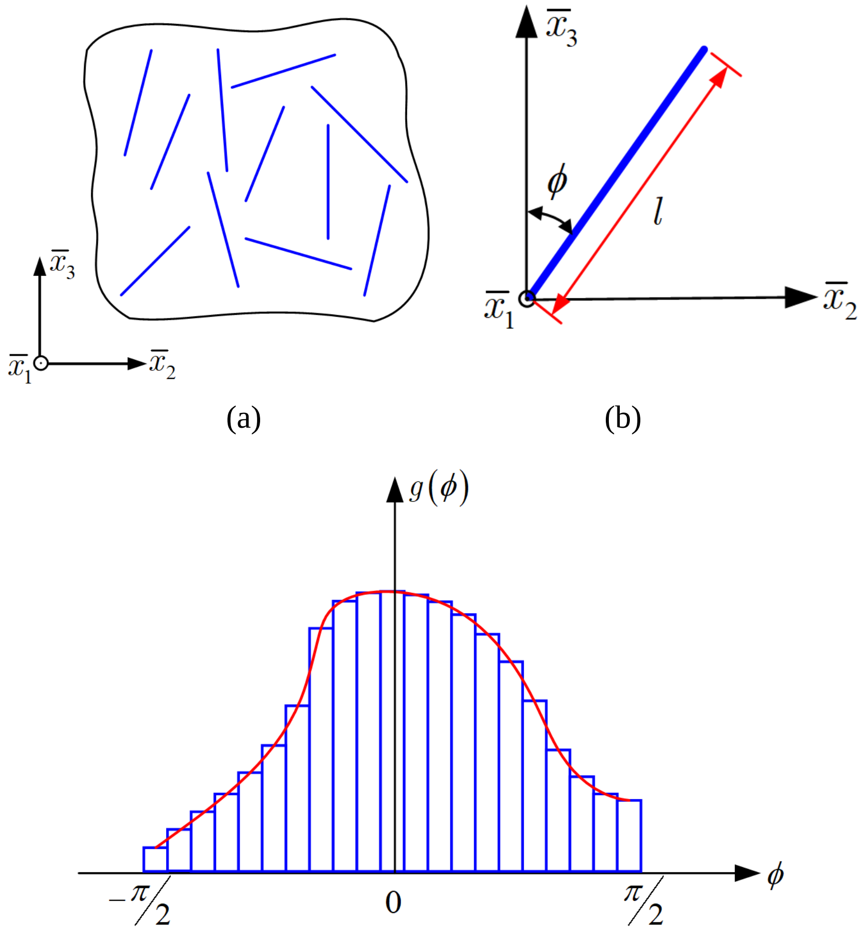
$$\varepsilon^i_1 = A^i_1 : \bar{\varepsilon} + A^i_{10} : \varepsilon^p_0 + \sum_{j=1}^{N} [A^{j,i}_{11} : \varepsilon^{p,j}_1 + A^{j,i}_{12} : \varepsilon^{p,j}_2], \qquad \text{(Eq. 3)}$$

$$\varepsilon^i_2 = A^i_2 : \bar{\varepsilon} + A^i_{20} : \varepsilon^p_0 + \sum_{j=1}^{N} [A^{j,i}_{21} : \varepsilon^{p,j}_1 + A^{j,i}_{22} : \varepsilon^{p,j}_2], \qquad \text{(Eq. 4)}$$

where the elastic and inelastic concentration tensors $A$ depend on the material properties and the corresponding interaction tensors.

The actual PA66/GF thin plate composite has a specific microstructure characterized by a well-defined skin-shell-core layer formation (Fig. 1b). In such microstructure, the fibers are oriented randomly following an orientation distribution function (ODF) $g(\varphi)$ (Fig. 2). With the help of this function, the secant stiffness tensor of the overall composite can be expressed as

$$C^{eff} = c_0 C_0 : A_0 + c_1 \int_{-\pi/2}^{\pi/2} g(\varphi) C_1(\varphi) : A_1(\varphi) d\varphi + c_2 \int_{-\pi/2}^{\pi/2} g(\varphi) C_2(\varphi) : A_2(\varphi) d\varphi, \quad \text{(Eq. 5)}$$




(c)

*Fig. 2. (a) Illustration of short-fiber reinforced composites with random fiber orientation; (b) The relative position of a fiber rotated by an angle of φ with respect to axis 3 on the 2 − 3 plane; (c) Approximation of the orientation density function using rectangles of constant width.*

where $C_0, C_1, C_2$ are the secant stiffness tensors of the phases, $c_0, c_1, c_2$ are the volume fractions of the phases and $A_0, A_1, A_2$ are the concentration tensors which depend on the material properties and the interaction tensors $T_r$ [2]. For practical purposes, the integrals are computed in a discrete form considering a specific number of fiber orientations.

The implementation of the modified Mori-Tanaka/TFA approach with randomly oriented fibers is achieved using the secant stiffness tensor and the return mapping algorithm technique [7]. For a known macroscopic strain increment, the stress overshoot is corrected through the inelastic stresses . The convergence of the numerical procedure is considered to be achieved when the difference of the macroscopic strains between two successive iterations is within a specified tolerance. It is worth noticing that the secant stiffness tensor approach is more advantageous than the more traditional tangential stiffness tensor approach in several aspects, notably in the integration of the damage mechanisms.

To calibrate and validate the designed model, a series of experimental tests have been carried out. Using available material properties for the PA66 matrix from the literature [8], the unknown parameters of the interphase region are identified through experiments on the composite. To this end, quasi-static tensile experimental tests have been conducted at two different strain rates on randomly oriented PA66/GF35 glass/polyamide specimens. The latter have been cut from the plate with different orientations (0°, 45°, 90°). The predictive capabilities of the micromechanics framework are demonstrated with the help of oligocyclic tests (Fig. 3). Moreover, comparison of numerical simulations with uniaxial tests on composites with different fiber content (PA66/GF30) illustrates the ability of the proposed approach to account for different microstructures.

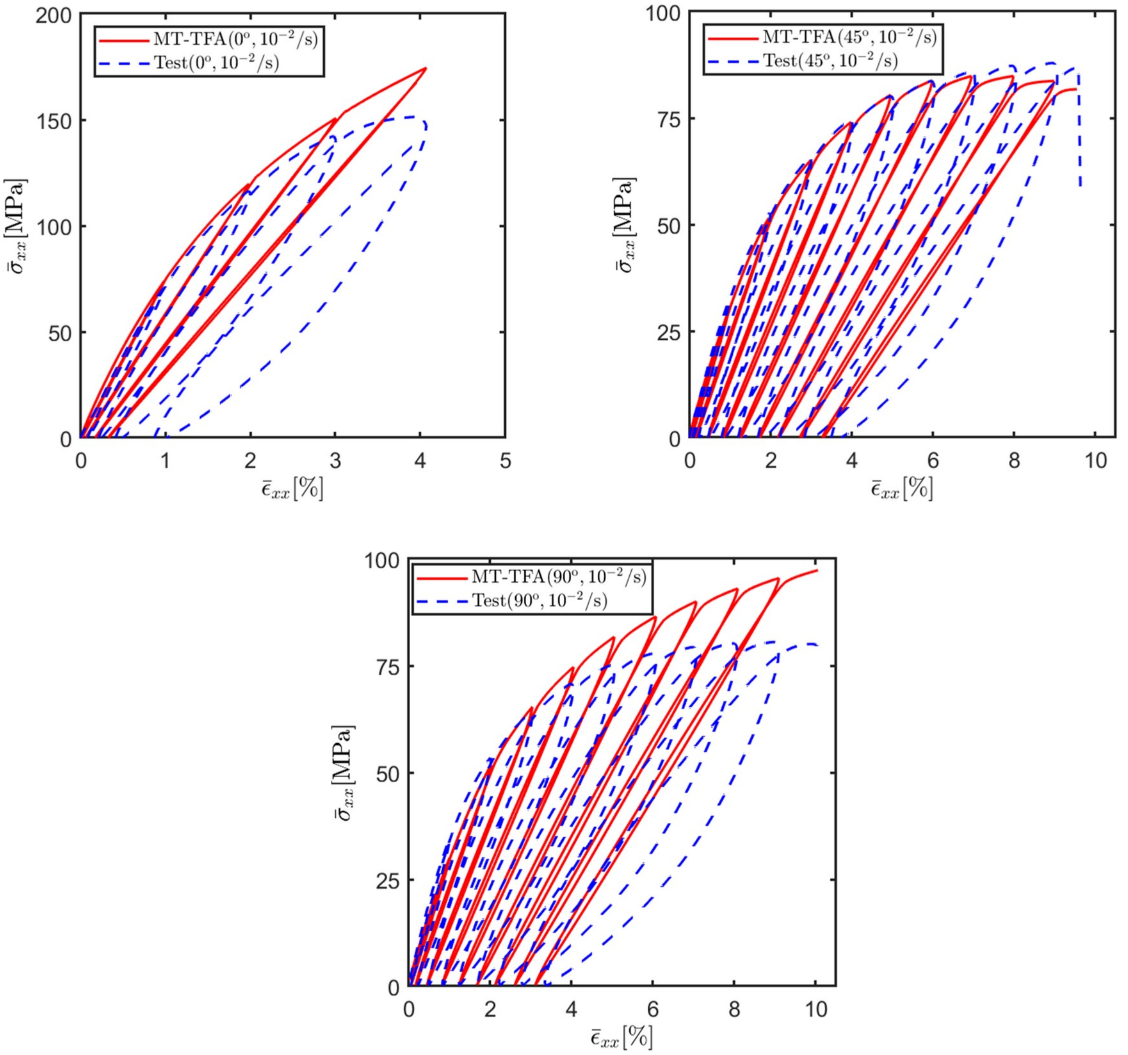
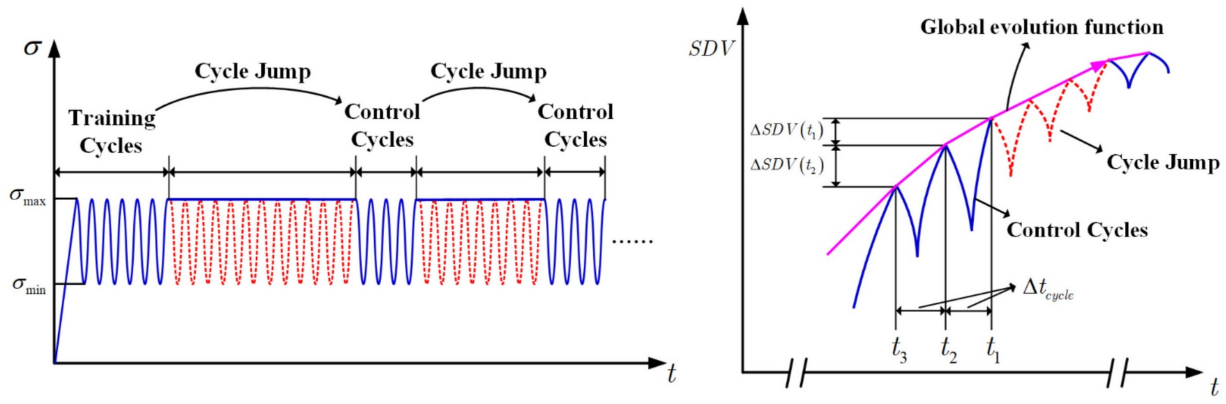



*Fig. 3. Comparison of the cyclic response of the 0°, 45° and 90° PA66/GF35 composites generated by the modified Mori-Tanaka/TFA approach against the experimental data at averaged loading rates of $10^{-2}$ /s.*

## 3. Combination of Mori-Tanaka/TFA with cycle jump technique

To capture the moderate and high cyclic response of PA66/GF short glass fiber composites, an accelerated micromechanics framework, combining the extended Mori-Tanaka/TFA framework and the cycle jump techniques is proposed. This extended theory accounts for microscopic viscoelastic-viscoplastic and damage mechanisms, and realistic microstructures formed during the injection mold process described by an orientation density function.

For the cycle jump, a number of training cycles are first conducted using the micromechanics scheme to obtain the global evolution functions of material state-dependent variables (SDVs) at each phase. These SDVs are extrapolated linearly to a certain jump length with the help of global evolution functions, allowing thus to skip a direct numerical simulation of several cycles. After the cycle jump, a set of complete cycles are performed based on the extrapolated SDVs using the Mori-Tanaka/TFA simulation to re-establish the global evolution functions.




*(a)* *(b)*

*Fig. 4. (a) Illustration of the cycle jump algorithm. (b) Approximation of the state-dependent variable evolution under cyclic loading.*

The implementation of the cycle jump procedure is facilitated by introducing an extrapolation control function to allow adaptive jump size control as well as minimize the extrapolating error [3]. The cycle jump procedure consists of the following steps (Fig. 4):

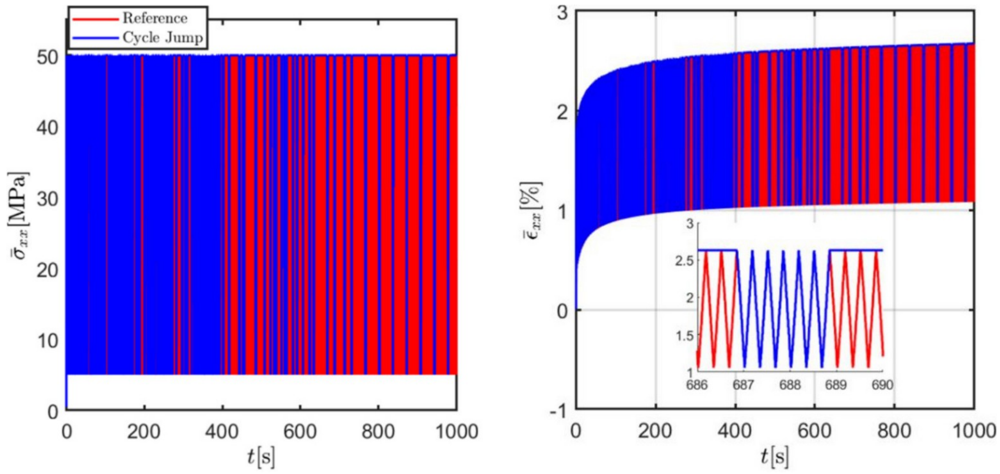



*Fig. 5. Comparison of the differences in applied stress and the strain response between cycle jump simulation andcycle-by-cycle reference solution at $45^0$ loading configuration.*

1) Training cycle: a number of training cycles are conducted using the Mori-Tanaka/TFA micromechanics scheme. These cycles provide the global evolution function of all the SDVs, which include the macroscopic stress, strain, inelastic strain, as well as viscoelastic strains, viscoplastic strains, damage variables in each phase.

2) Cycle jump: the maximum common jump length of all the SDVs is established. Then the SDVs are extrapolated based on the global evolution function to a certain jump length.

3) Control cycle: the extrapolated SDVs are utilized as the initial state for obtaining a new set of the Mori-Tanaka/TFA micromechanics simulations after the cycle jump. The global evolution functions of all the SDVs are updated according to the control cycle analysis.

4) Repeat steps 2)-3) to obtain the response till the desirable number of cycles or time.

Fig. 5 illustrates the comparison of the applied stress-time and the strain-time response between cycle jump simulation and cycle-by-cycle reference solution for PA66/GF specimen at 45 deg orientation. It is observed that the cycle jump solutions show good accordance with the cycle-by-cycle reference solution. The proposed extrapolation technique enables automatic determination of a suitable jump length during the cyclic loading. Accordingly, when there are fast variations in the SDVs, such as in the initial portion of loading cycles where significant nonlinearity occurs, the Mori-Tanaka/TFA computations are conducted at shorter or no jumps. When the SDVs are stabilized, the extended micromechanics approach performs longer jumps, thereby eliminating the need for extensive numerical computation during the period covered by the cycle jump.

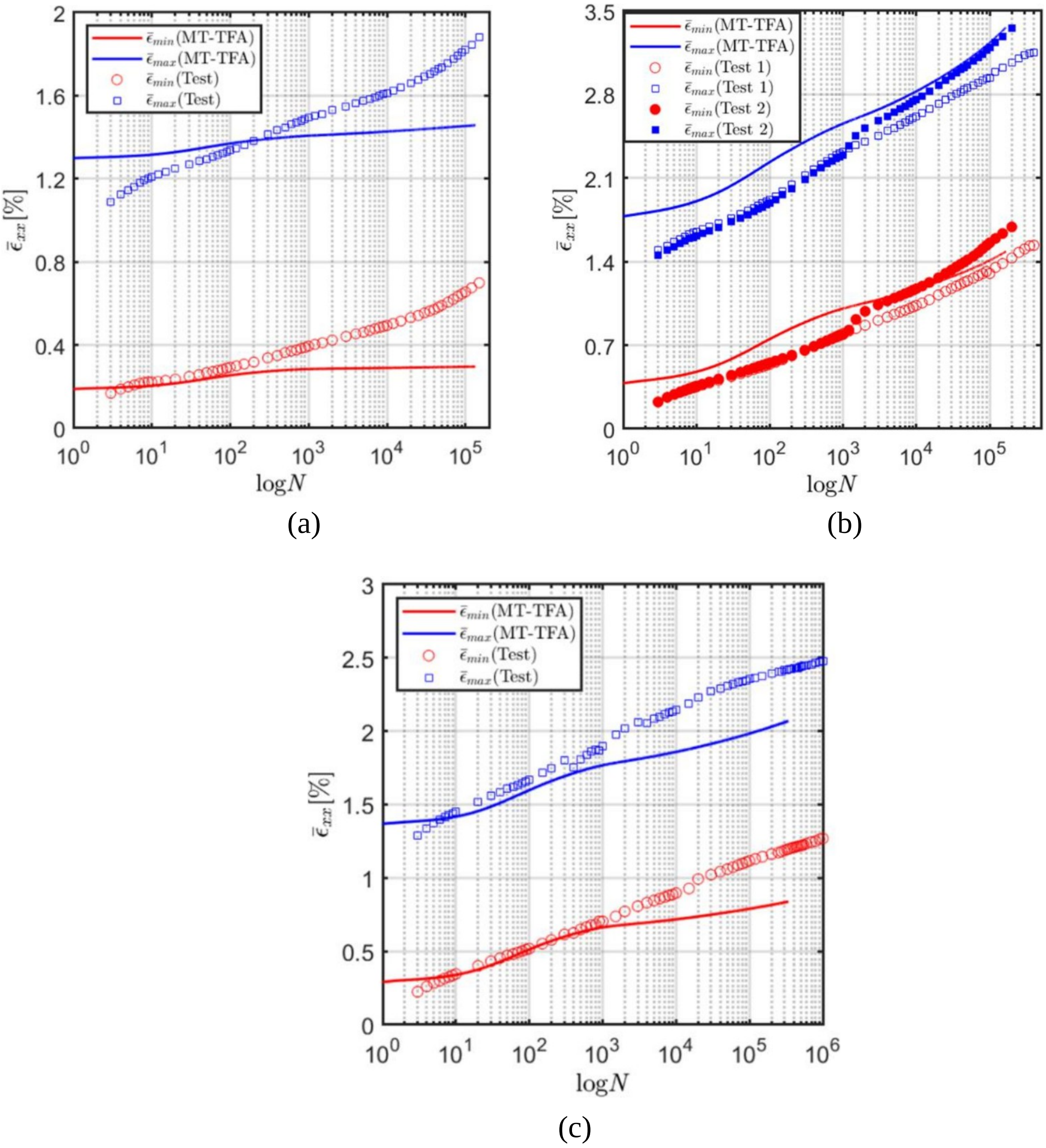



*Fig. 6. Comparison of the maximum and minimum strains at each loading cycle for PA66/GF30 composites: (a) 0 deg under R=0.1, $\sigma_{max}$=80MPa. (b) 45 deg under R=0.1, $\sigma_{max}$=50MPa. (c) 90 deg under R=0.1, $\sigma_{max}$=40MPa.*

To demonstrate the methodologies capacities, numerical simulations are compared with experimental results for three orientations (0°, 45° and 90°). Fig. 6 shows the minimum and maximum strains generated during a cyclic loading at specific stress ratios ($R=\sigma_{min}/\sigma_{max}$). The results are within acceptable error ranges, making the proposed approach suitable to capture the cyclic response of PA66/GF composites. Nevertheless, a close examination of Fig. 6 reveals that greater differences may occur at high loading cycles. This is particularly true for the specimen loaded with high mean stress. The reasons for these discrepancies at high cycle range and high mean stresses are several-fold. First of all, the viscoelastic parameters of the polyamide phase have been identified from experiments at low characteristic time, thus can only represent the short or medium-term response. In order to capture long-term fatigue creep, higher characteristic times should be included in the model through additional viscoelastic branches. Secondly, the experimental responses manifest important "banana effect" characterized by larger hysteresis loops with increasing loading cycles. This effect is induced by the friction of the cracked phases combined with the viscoelasticity nature of the polyamide, which the extended Mori-Tanaka/TFA method does not capture. Thirdly, high cycle experiments exhibit significant variability at high loading cycles. The relative humidity and temperature of the specimens during the full cycle test changes remarkably due to the energy dissipation and self-heating under the high-cycle range. These effects are currently not taken into account in the proposed micromechanics scheme.

## 4. Conclusions

This work proposes a mean-field micromechanics scheme for polyamide based randomly oriented short glass fiber composites, considering the Mori-Tanaka theory and the Transformation Field Analysis approach. The numerical implementation of this homogenization approach is performed in the form of a "macroscopic constitutive law" that can be integrated in structural applications. Validation of the model with experimental results reveals its capabilities. Moreover, a cycle jump-aided micromechanics methodology, combined with the extended Mori-Tanaka/TFA method has been developed for simulating the high-cycle viscoelastic-viscoplastic response of PA66/GF composites with ductile damage. This novel framework circumvents the computation challenges imposed by the cycle-by-cycle simulation during the high cycle loading. It simulates only a minimum number of cycles at the chosen time intervals while most of the cycles between these intervals are skipped using linear extrapolation. The implementation of the cycle jump technique is further facilitated by introducing an adaptive extrapolation scheme, which permits not only the automatic control of cycle jump length but also good accuracy of the accelerated simulations.

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