MATHEMATICAL ISSUES IN MECHANICAL TOLERANCE ANALYSIS.

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Résumé:

The aim of this paper is to provide an overview of tolerance analysis. Tolerancing decisions can profoundly impact the quality and cost of product. There is a strong need for increased attention to tolerance design to enable high-precision assemblies to be manufactured at lower costs. Indeed, tolerance analysis is a key element in industry for improving product quality. Designers want tight tolerances to assure product performance; manufacturers prefer loose tolerances to reduce cost. There is a critical need for a quantitative design tool for specifying tolerances. Tolerance analysis brings the engineering design requirements and manufacturing capabilities together in a common model, where the effects of tolerance specifications on both design and manufacturing requirements can be evaluated quantitatively. Significant amount of literature is related to tolerancing methods. Summaries of state of the art, the most recent developments, and the future trends in tolerancing research can be found. This paper provides a classification of the issues from a mathematical point of view.

Mots clés: Tolerance analysis, Mathematical issues, Overview

1 Introduction

As technology increases and performance requirements continually tighten, the cost and the required precision of assemblies increase as well. There is a strong need for increased attention to tolerance design in order to enable high-precision assemblies to be manufactured at lower costs. Due to the variations associated with manufacturing process, it is not possible to attain the theoretical dimensions in a repetitive manner. It causes a degradation of functional characteristics of the product. In order to ensure the desired behavior and the functional requirements of the system in spite of variations, the component features are assigned a tolerance zone within which the value of the feature i.e. situation and intrinsic lie.

Therefore, tolerance analysis is a key element in industry for improving product quality and decreasing the manufacturing cost. In addition, it participates to an eco-aware attitude since it allows industrials to manage and reduce scrap in production. Tolerance analysis concerns the verification of the value of functional requirements after tolerance has been specified on each component. Currently, this verification is totally dependent on the models chosen before. Currently, trial runs or very simple simulation models (1D linear tolerance charts for example) are used to check the quality criterion. This approach can be called into question: the trial runs are very costly and time consuming. Researchers have recognized the inefficiency of such simple simulation models based on explicit system response function which represents the variation accumulation. For complex systems, determination of explicit system response function is very complex, whereas this determination is easy for an open kinematic chain without gap. Research efforts have been devoted to developing an efficient simulation model for tolerance analysis.

Currently, the developed approaches depend on the type of geometrical model and on the type of system response function or simulation model (behavior model). Therefore, their scopes are limited and some problems are not addressed. Moreover, the industrial practices are based on the decomposition of the system kinematic configurations and the simplification of the system response function which are not efficient.

This paper deals with some issues of mechanical tolerance analysis. It presents an overview for improving tolerance analysis which is the cause of the ANR Project AHTOLA (Advanced Hybrid method for the TOLerance Analysis of complex system). In the following text, the some of the limitations of the common engineering approaches for tolerance analysis are pointed out. Also, a classification of tolerance analysis problems is proposed based on the mathematical point of view.

2 CONTEXT

As technology moves forward beyond the industrial economy to the information and knowledge based economies, the EU and particularly France has an opportunity to recapture the lost ground and assume leadership in the new wave of technologies, tools and processes; foundations of a new generation of manufacturing. To achieve this, it is imperative to develop approaches in order to enable high-precision systems to be manufactured at lower costs.

Today, most of the products are developed using CAx software in Concurrent Engineering (CE) context. Concurrent Engineering (CE) is an engineering and management philosophy, to the integrated, concurrent design of products and their processes; including manufacturing and support. This approach is intended to cause the developers, from the outset to consider all elements of the product lifecycle from conception through disposal, including quality, cost, schedule and user requirements [33]. This approach demands the formation of a cross-functional product development team, which includes people from a wide range of departments, such as: product planning, design, manufacture, assembly, quality assurance, marketing, sales and finance.

Dimensions and tolerances influence almost all aspects of product development which are of interest to CE team members who consider all the life cycle issues of a product during its design stage. Integration of tolerance analysis therefore becomes and inevitable activity of the concurrent engineering process that affects, the product quality, cost and performance. Therefore, a CE approach will be ideal for the selection of dimensions and tolerances through applications of FD&T methodology. Furthermore, FD&T can serve as a common link between all members of the CE team; hence it can enhance the CE team performance.

Srinivasan mentions that *"We may consider the ubiquitous tolerances in various stages of a product life cycle. Since the role of tolerances in a life cycle varies from stage to stage, depending on their own respective objectives, it is not a trivial task to take all these different factors into account when a designer determines a tolerance." [37]*.

Moreover, Chase mentions that *"Both engineering design and manufacturing are concerned with the magnitude of tolerances specified on engineering drawings, as shown in figure 1."[7]*

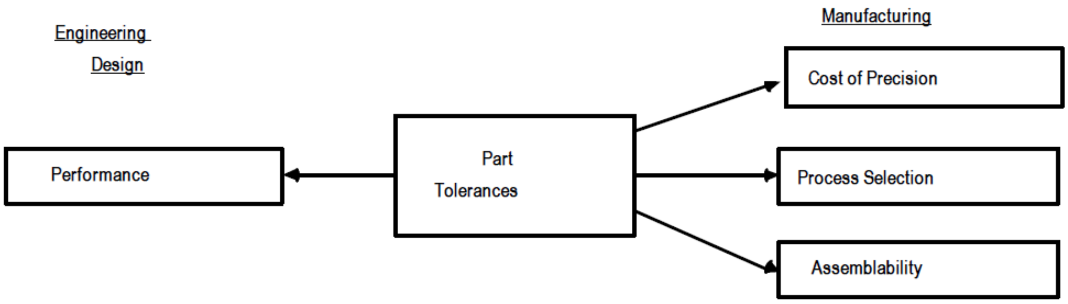

Figure 1. Effects of part tolerances

Variation/Uncertainty is ubiquitous in any engineering system at any stage of product development and throughout a product life cycle. Variation/Uncertainty is an unavoidable reality of the engineering. Managing the Variation/Uncertainty is a crucial part of any engineering activity and plays a fundamental role in the success of the product under design. Such Variation/Uncertainty has a significant impact on product performance. The product performance improvement with several Variation/Uncertainty types is very important to avoid warranty returns and scraps. Variation and uncertainty may arise in experiments, processing methods, material structure, and model parameters that support concurrent design of materials and products/systems. Potential sources of Variation/Uncertainty in a system also include human errors, manufacturing or processing variations, variations of operating conditions, inaccurate or insufficient data, model assumptions and idealizations, microstructure variability, and lack of knowledge. One of main sources of uncertainty and variation in the case of mechanical systems consisting of mating or interconnected sub components is the variations arising from manufacturing processes. The ability to design components in presence of variation while keeping their ability to perform as per functional requirements is the goal of tolerancing. Tolerancing is therefore an important part of the design process and a key component that must be addressed if the robustness of the product is to be ensured. It is therefore imperative that tolerancing activity be accounted for in the earliest possible phase of product design.

Usually, the used approach for tolerance analysis of a complex system like gear pump is based on experimentations. In order to determine the effects of a tolerance and to understand the contributions of tolerances on the system behavior, it is necessary to identify the relationships between tolerances and functional characteristics by a set of experiments. This approach is expensive and not very flexible.

To improve the tolerancing process in an industrial context, there exists a strong need for tolerance analysis to estimate the ppm (defected product per million – probability of non-quality) with high-precision computed at lower cost. The engineers need tolerance analysis methods:

- to improve product quality,
- to decrease the manufacturing cost,
- to reduce scrap in production (eco-aware attitude), and customer returns (Toyota event – 26 January 2011 : return of 1.7 million vehicles).

3 BRIEF STATE OF THE ART

The tolerancing decision should respect the limited capabilities of the required manufacturing processes as well as the functionality and/or assemblability constraints. The ubiquity of tolerances entails the various tolerance related problems in different stages of life cycle, characterized by their respective objectives and viewpoints. These individual problems are interrelated with each other, which makes the tolerancing research more challenging to be handled efficiently. In [27], the authors have classified the tolerance-related research into seven distinct categories: Tolerance schemes;

Tolerance modeling and representation; Tolerance specification; Tolerance analysis; Tolerance synthesis or allocation; Tolerance transfer; Tolerance evaluation.

This paper focuses on Tolerance analysis which is needed for Tolerance synthesis, Tolerance transfer and Tolerance evaluation.

Shen mention that*:"The objective of tolerance analysis is to check the extent and nature of the variation of an analyzed dimension or geometric feature of interest for a given GD&T scheme. The variation of the analyzed dimension arises from the accumulation of dimensional and/or geometrical variations in the tolerance chain"* [36].

Usually, tolerance analysis can be either worst-case or statistical:

- Worst-case analysis (also called deterministic or high-low tolerance analysis) involves establishing the dimensions and tolerances such that any possible combination produces a functional assembly, i.e. the probability of non-assembly is identically equal to zero. It considers the worst possible combinations of individual tolerances and examines the functional characteristic. Consequently, worst-case tolerancing can lead to excessively tight part tolerances and hence high production costs [27].

- Statistical tolerancing is a more practical and economical way of looking at tolerances and works on setting the tolerances so as to assure a desired yield. By permitting a small fraction of assemblies to not assemble or function as required, an increase in tolerances for individual dimensions may be obtained, and in turn, manufacturing costs may be reduced significantly. Statistical tolerance analysis computes the probability that the product can be assembled and will function under given individual tolerance [34] .

There are three main issues in tolerance analysis:

1. The models for representing the geometrical deviations,

2. A mathematical model for calculating the system behavior with deviations,

3. The development of the solution techniques or analysis methods. Such as worst-case searching and statistical analysis.

This paper focuses on the last issue. Based on the mathematical point of view, the tolerance analysis methods could be divided into two distinct categories: displacement accumulation or tolerance accumulation.

- The aim of displacement accumulation is to simulate the influences of deviations on the geometrical behavior of the mechanism. Usually, tolerance analysis uses a relationship of the form [34]: $Y= f(x_1, x_2, ..., x_n, g_1, g_2, ..., g_m)$ where Y is the response (characteristic such as gap or functional characteristics) of the assembly, $x_1, x_2, ..., x_n$ are the values of some characteristics (such as situation deviations or/and intrinsic deviations) of the individual parts or subassemblies making up the assembly, and $g_1, g_2, ..., g_m$ are the values of gaps. The part deviations could be represented by kinematic formulation [32], [15], [16], small displacement torsor (SDT) [4], [30], matrix representation [7], vectorial tolerancing [21], … The function f is the assembly response function which represents the deviation accumulation. It could be an explicit analytic expression, an implicit analytic expression, or numerical simulation for which it is possible to compute a value for Y given values of $x_1, x_2, ..., x_n$ and $g_1, g_2, ..., g_m$. In a particular relative configuration of parts of an assembly consisting of gaps without interference between parts, the composition relations of displacements in some topological loops of the assembly permits to determine the function *f*. For complex system, determination of explicit function *f* is very complex [1], [8].

- The aim of tolerance accumulation is to simulate the composition of tolerances i.e. linear tolerance accumulation, 3D tolerance accumulation. Based on the displacement models, several vector space models map all possible manufacturing variations (geometrical displacements between manufacturing surfaces or between manufacturing surface and nominal surface) into a region of hypothetical parametric space. The geometrical tolerances or the dimensioning tolerances are represented by deviation domain [24], [25], T-Map® [2] or specification hull [8], [9], [13]. These three concepts are a hypothetical Euclidean volume which represents all possible deviations in size, orientation and position of features. For tolerance analysis, above mathematical representations of tolerances allow calculation of accumulation of the tolerances by Minkowsky sum of deviation and clearance domains [24], [38]; to calculate the intersection of domains for parallel kinematic chain; and to verify the inclusion of a domain inside other one. The methods based on this mathematical representation of tolerances are very efficient for the tolerance analysis of linear models.

Current methods are believed to have major drawbacks that reduce the accuracy of tolerance stack-up evaluation. These drawbacks are:

- The limited scope of the statistical approaches: explicit functions without gap and numerical simulations without gap. To use a statistical approach, it needs to simplify the mechanical model of a hyperstatic system with gaps. This simplification is the current industrial practice.

- The limited scope of the tolerance accumulation approaches: linear problem (linear accumulation by Minkowsky sum).

Therefore, a new formulation of the tolerance analysis based on the quantifier notion [8] was developed:

- the mathematical expression of tolerance analysis for assembly requirement is: *“For all acceptable deviations (deviations which are inside tolerances), there exists a gap configuration such as the assembly requirements and the behavior constraints are verified”*.

- the mathematical expression of tolerance analysis for functional requirement is: *“For all acceptable deviations (deviations which are inside tolerances), and for all admissible gap configurations, the assembly and functional requirements and the behavior constraints are verified”*.

The quantifiers $\forall$ and $\exists$ provide a univocal expression of the condition corresponding to a geometrical product requirement. This opens a wide area for research in tolerance analysis. To compute this mathematical formulation, two approaches based on Quantified Constraint Satisfaction Problem (QCSP) solvers and Monte Carlo simulation are proposed and tested in the cases of implicit functions [35], [8], [26] and numerical simulations [5], [10] for behavior analysis. But the expressive power of QCSP doesn’t enable to model all mathematical formulations of tolerance analysis. Monte Carlo simulation has been used in conjunction with the quantifier notion to calculate the probability of all requirements. For the consistent and reliable application of the Monte Carlo simulation to the statistical tolerance analysis, the number of samples is the key of precision. By a large number of samples, the precision can be improved, but the computational cost will be increased. The improvement of this approach should be an area for some intense research on stochastic methods coupled with worse case methods.

The following sections of the state of art focus on the stochastic methods for tolerance analysis.

For statistical tolerance analysis, the input variables $X_1,...,X_n$ are continuous random variables which enable to represent the imperfections and tolerances. In general, they could be mutually dependent. In the field of structural reliability analysis lots of methods exist [17], [31] for the evaluation of the so-called probability of failure i.e. the probability that a structure, mechanism, … does not satisfy the mechanical resistance requirement $\text{Prob}\ f(X_1,...,X_n) \leq s$, s being a threshold value and f the function that characterizes the structural behavior.

In this field, lots of methods have been developed to consider analytical function *f*. The simulation techniques such as Monte Carlo simulation remain the reference method but require lots of mechanical computations that makes it very difficult to use in practice for industrial applications. In the field of tolerance analysis, [34], [39] use this method on very simple stack up without gap.

In the last decade, the increasing interest of accurate but time consuming numerical methods, such as the Finite Element Methods for the prediction of mechanical behavior, has involved the development of approximated probabilistic methods. First the FORM/SORM methods [26] provide an approximation of the probability with a linearization of the *f* function around the most probable failure point. These types of methods were already applied recently in the framework of analytical tolerancing problem without gap [23]. Next, Response Surface Method [22] and chaos polynomial methods [3], based on a higher order polynomial *f* function expansion, try to decrease the time consuming of the probability computation. Lots of approximated methods based on the most probable failure point have the disadvantage to consider only one main failure scenario. For system reliability problems with more than one scenario (it is the case when dealing with problem with gaps), FORM system methods based on the use of the multi-dimensional Gaussian distribution can provide good approximation of the failure probability. In addition, multi-FORM methods can give approximation of the probability of failure finding all the most probable failure points of the problem.

More recently, various authors aim at using more precise meta-modeling such as Support Vector Machine [14] or Kriging method [18] in order to get a precise estimation of probability with the less possible number of *f* evaluations. The interest of such methods is to use classification methods in order to separate safe and unsafe domain [14] or safe and unsafe points of the Monte Carlo simulation [18]. These methods based on appropriate meta-models have the advantage to treat very time consuming mechanical models, sometimes highly nonlinear with local minimums, …

Considering the probability computation of mechanical systems with gaps, the literature on that subject seems to be very poor. Only two references seem to be available [1], [8]. In [1], the authors tried to consider the problem using system reliability methods from hypothesis on contact points. This approach could be an interesting direction for further investigation. In [8], the authors focus on the way to find the worst gap, the probability computation being achieved by Monte Carlo simulations.

4 CLASSIFICATION OF ISSUES & UNIFIED MATHEMATICAL FORMULATION.

The previous section details a brief state of art, and some limitations are pointed out. In this section, we propose a classification of issues of tolerance analysis based on the type of the behavior model with deviations. The behavior model is the assembly response function which represents the deviation accumulation. It could be an explicit analytic expression, an implicit analytic expression, or numerical simulation for which it is possible to compute a value for some functional characteristics given values of part deviations and gaps.

Tolerance analysis concerns the verification of the value of functional requirements after tolerance has been specified on each component. To do so, it is necessary to simulate the influences of component deviations on the geometrical behavior and the functional characteristics of the mechanism. The geometrical behavior model needs to be aware of the surface deviations of each component (situation deviations and intrinsic deviations) and relative displacements between components according to the gap. The model used in this paper is a parameterization of deviations from theoretic geometry, the real geometry of parts is apprehended by a variation of the nominal geometry.

The deviation of component surfaces, the gaps between components and the functional characteristics are described by parameters:

- $X=\{x_1, x_2, \ldots., x_n\}$ are the parameters which represent each deviation (such as situation deviations or/and intrinsic deviations) of the components making up the mechanism.

- $G=\{g_1, g_2, \ldots., g_m\}$ are the parameters which represent each gap between components

In the case of analytic formulation, the mathematical formulation of tolerance analysis takes into account the influence of geometrical deviations on the geometrical behavior of the mechanism and on the geometrical product requirements; all these physical phenomena are modeled by constraints on the parameters:

- $C_c(X,G)= 0$: Composition relations of displacements in the various topological loops express the geometrical behavior of the mechanism. They define compatibility equations between the deviations and the gaps. The set of compatibility equations, obtained by the application of composition relation to the various cycles, makes a system of linear equations. So that the system of linear equations admits a solution, it is necessary that compatibility equations are checked.

- $C_i(X,G) \leq 0$ and $C_{i^*}(X,G) = 0$: Interface constraints limit the geometrical behavior of the mechanism and characterize non-interference or association between substitute surfaces, which are nominally in contact. These interface constraints limit the gaps between substitute surfaces. In the case of floating contact, the relative positions of substitute surfaces are constrained technologically by the non-interference, the interface constraints result in inequations. In the case of slipping and fixed contact, the relative positions of substitute surfaces are constrained technologically in a given configuration by a mechanical action. An association models this type of contact; the interface constraints result in equations.

- $C_f(X,G) \leq 0$: The functional requirement limits the orientation and the location between surfaces, which are in functional relation. This requirement is a condition on the relative displacements between these surfaces. This condition could be expressed by constraints, which are inequations.

Mechanism can be divided into two main categories in terms of degree of freedom: Iso-constrained mechanisms, and over-constrained mechanisms. Given their impact on the mathematical formulation for the problem of tolerance analysis, a brief discussion of these two types is given by Ballu et al. [1]:

- *“Isoconstrained mechanisms are quite easy to grasp. Geometrical deviations within such products do not lead to assembly problems; the deviations are independent and the degrees of freedom catch the deviations. When considering small deviations, functional deviations may be expressed by linear functions of the deviations.”*

- *"Considering overconstrained mechanisms is much more complex. Assembly problems occur and the expression of the functional deviations is no more linear. Depending on the value of the manufacturing deviations:*

    - *the assembly is feasible or not;*

- *the worst configuration of contacts is not unique for a given functional deviation.*

For each overconstrained loop, events on the deviations have to be determined:

- *events ensuring assembly,*
- *events corresponding to the different worst configurations of contacts.*

As there are different configurations, the expression of the functional deviation cannot be linear."

Therefore, in the case of analytic formulation for isoconstrained mechanisms or for simple overconstrained mechanism, it is possible to transform the previous formulation into an explicit function f which is the assembly response function: $Y=f(X)$ where Y is the response (characteristic such as gap or functional characteristics) of the assembly.

In some cases, the geometrical deviations impact some non-geometrical functional requirements. To simulate the influences of geometrical deviations on these requirements, an analytic formulation cannot possibly be employed. To do so, it is necessary to use numerical simulation for which it is possible to compute a value for Y given values of deviations and gaps: $Y=f_{numerical\ simmilation}(X)$ or $Y=f_{numerical\ simmilation}(X,G)$

In summary, the Figure 2 illustrates the issue classification and the link between these issues and the identified approaches in the previous section.

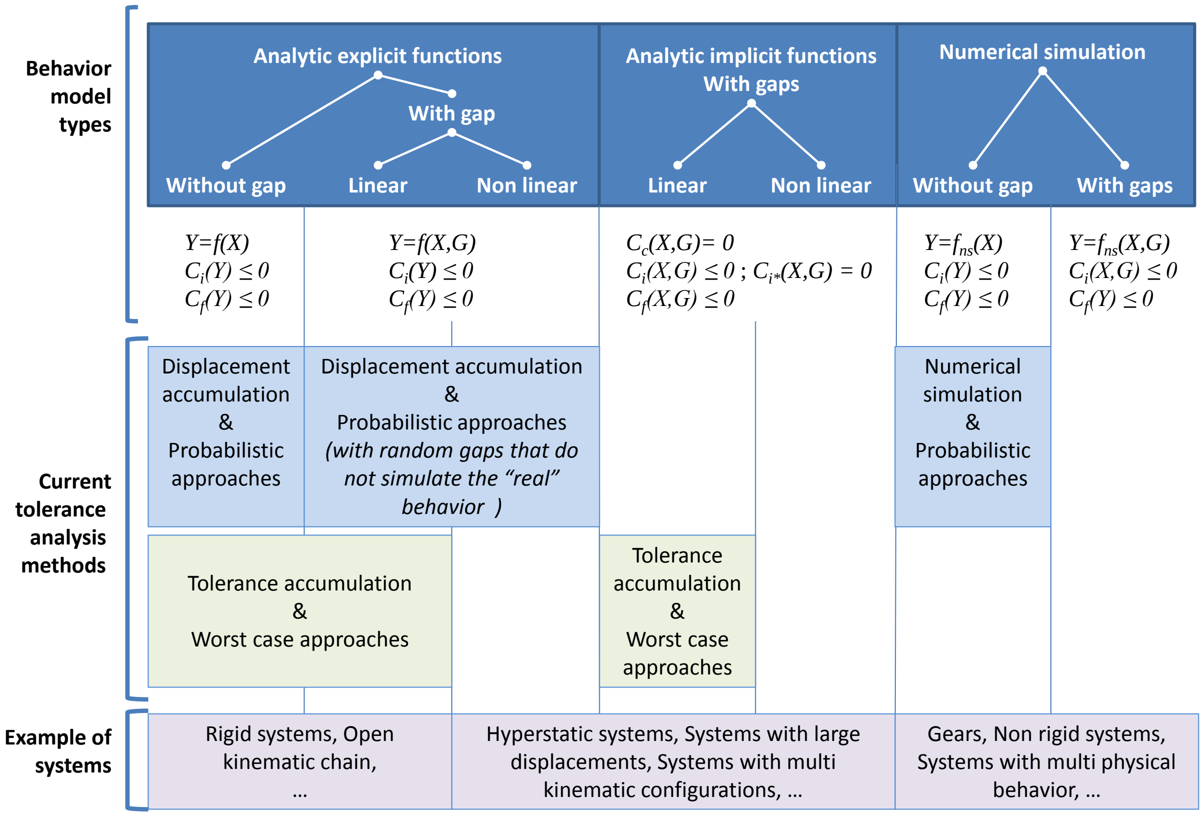

Figure 2. Overview of issues and tolerance analysis approaches.

In the following section, this paper focuses on statistical tolerance analysis. And, to improve the tolerancing process in an industrial context, there exists a strong need for statistical tolerance analysis to estimate the probability expressed in ppm (defected product per million) with high-precision computed at lower cost. Two probabilities are considered:

- P_A: the probability of the assemblability for a given tolerance specification. Let AC be the event that the assemblability condition for a given assembly is respected. The condition of the assemblability describes the essential condition for the existence of gaps that ensure the assembly of the components in the presence of the part deviations. In order for a mechanism to assemble successfully, the different components in the presence of deviations should assemble without interference and should have a specific set of gaps that characterize the instance of the assembly. This condition stipulates the use of an existential quantifier for an initial search for the existence of a feasible configuration of gaps: *"there exists an admissible gap configuration of the mechanism such that the assembly requirement (interface constraints) and the compatibility equations are respected" (Assemblability condition)*.

- P_{FR}: the probability of respect of the functional requirements. Let FC be the event that the functional condition are fulfilled. Once a mechanism assembles, in order to evaluate its performance under the influence of the deviations, it is necessary to describe an additional condition that evaluates its core functioning with respect to the basic product requirements. In terms of the tolerance analysis, the basic requirement becomes the maximum or minimum clearance on a required feature that would have an impact on the mechanism's performance. The most essential condition therefore becomes that for all the possible gap configurations of the given set of components that assemble together, the functional condition imposed must be respected. In terms of quantification needs, in order to represent all possible gap configurations, the universal quantifier is required: *"for all admissible gap configurations of the mechanism, the geometrical behavior and the functional requirement are respected" (functional condition).*

The probability expression of the two conditions is detailed in the Table 1:

Analytic explicit expression without gap $P_A = P(AC) = P(C_i(f(X)) \le 0)$ $P_{FR} = P(FC) = P(C_f(f(X)) \le 0)$
Analytic explicit expression with gaps $P_A = P(AC) = P(C_i(f(X,G)) \le 0)$ G is considered as free parameters $P_{FR} = P(FC) = P(C_f(f(X,G)) \le 0, \forall G \in \{G \in \boldsymbol{R}^m : C_c(X,G)= 0 \cap C_i(X,G) \le 0 \cap C_{i*}(X,G) = 0\})$
Analytic implicit expression with gaps $P_A = P(AC) = P(C_c(X,G)= 0 \cap C_i(X,G) \le 0 \cap C_{i*}(X,G) = 0)$ G is considered as free parameters $P_{FR} = P(FC) = P(C_f(X,G) \le 0, \forall G \in \{G \in \boldsymbol{R}^m : C_c(X,G)= 0 \cap C_i(X,G) \le 0 \cap C_{i*}(X,G) = 0\})$
Numerical simulation without gap $P_A = P(AC) = P(C_i(f_{ns}(X)) \le 0)$ $P_{FR} = P(FC) = P(C_f(f_{ns}(X)) \le 0)$
Numerical simulation with gaps $P_A = P(AC) = P(C_i(f_{ns}(X,G)) \le 0)$ G is considered as free parameters $P_{FR} = P(FC) = P(C_f(f_{ns}(X,G)) \le 0, \forall G \in \{G \in \boldsymbol{R}^m : C_c(X,G)= 0 \cap C_i(X,G) \le 0 \cap C_{i*}(X,G) = 0\})$

Table 1. Probability expressions.

Comparing to classical probability assessment, the main scientific challenge concerns the development of approaches to estimate these probabilities with gaps. In addition to this, the second challenge is to evaluate the probability computation in an acceptable computing time and managing the accuracy of the results.

5 Conclusion

Research into tolerancing in the field of engineering has been significant, particularly in the last two decades. The first part of the paper provided a context and a brief state of art on the mathematical approaches for tolerance analysis. The outcomes of this part are the overview of mathematical aspect of tolerance analysis approaches, and some issues which are due to the limitations of the tolerance analysis approaches. The last part of the paper provided an issue classification based on the mathematical point of view, and some scientific challenges. One of the most important challenges is the development of approaches to estimate these probabilities with gaps. It should be an area for some intense research.

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