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## **HIGH SPEED FUNCTIONALITY OPTIMIZATION OF FIVE-PHASE PM MACHINE USING 3<sup>RD</sup> HARMONIC CURRENT**

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**Abstract.** Some surrogate-assisted optimization techniques are applied in order to improve the performances of a 5-phase Permanent Magnet (PM) machine in the context of a complex model requiring computation time. An optimal control of four independent currents is proposed in order to minimize the total losses with the respect of functioning constraints. Moreover, some geometrical parameters are added to the optimization process allowing a co-design between control and dimensioning. The effectiveness of the method allows solving the challenge which consists in taking into account inside the control strategy the eddy-current losses in magnets and iron. In fact, magnet losses are a critical point to protect the machine from demagnetization in flux-weakening region. But these losses, which highly depend on magnetic state of the machine, must be calculated by Finite Element Method (FEM) to be accurate. The FEM has the drawback to be time consuming. It is why, a direct optimization using FEM is critical. The response surface method (RSM) and the Efficient Global Optimization (EGO) algorithm consist in approximating the FEM by a surrogate model used directly or indirectly in the optimization process. The optimal results proved the interest of the both methods in this context.

**Keywords:** surrogate-assisted optimization, five-phase PM machine, flux weakening

### **INTRODUCTION**

Multiphase drives are used in different areas, such as electrical ship propulsion [1], aerospace [2] and hybrid-electric vehicles [3]. Compared to the traditional 3-phase drives, they present specific advantages: tolerance to faults especially coming from power electronics devices; lower pulsating torque; splitting the power across more inverter legs especially for very high power drives or for 10-15 kW very low voltage (<60V) drives in automotive sector. Moreover in comparison with three-phase drives, supplementary degrees of freedom that are favorable to optimization appear concerning the current control [4]. In this paper, a five-phase machine, designed for automotive applications [5], is considered. This machine presents fractional-slot concentrated windings because of their high torque/volume ratio, high efficiency, and simple winding structure [6]. However, high rotor losses (in magnets and iron) are one of the undesired parasitic effects which can appear with such kind of machine windings because of high level of space harmonics, whose impact is particularly significant at high speed in the flux weakening zone [7]-[9]. These rotor losses reduce the efficiency of the machine and furthermore they can cause magnet heating which increases the risk of magnet demagnetization, leading finally towards full breakdown. Researches have been done in order to develop an optimal flux weakening strategy (choosing the optimal current vector) in 3-phase PM machines [10]-[12] and a few one for multiphase machines [13]-[16]. In the cited researches, copper losses are always the first criteria to be minimized while iron and magnet losses are not taken into account. The reason of this absence is the lack of accurate analytical model for the calculation of the eddy-current losses and the necessity to have a finite element model to calculate them. As consequence the corresponding optimizations are only reliable for low speeds and with classical integral slot winding machines whose MagnetoMotive Force (MMF) is clean from harmful parasitic harmonics. In [17] an optimization is done for three-phase machine taking into account copper losses and iron losses using FEM software.

The present paper concerns the design of a 5-phase wye-coupled high speed drive. For a 5-phase machine, it is possible to obtain torque with only the first harmonic currents in order to produce the torque. Only two degrees of freedom are then necessary. It remains then 2 degrees of freedom since 4 independent currents can be imposed. The two remaining degrees of freedom can be used in order to optimize either the required power for the Voltage Source Inverter (VSI) or the losses in the machine. It can be shown that in case of no third harmonic electromotive force, the injection of third harmonic current has no impact on the torque but has an effect on the machine losses or on the power delivered by the VSI. In this paper, the objective is to maximize the efficiency of low voltage five-phase machine with concentrated windings considering iron and also magnet losses, in which the fundamental current, the 3<sup>rd</sup> harmonic current and two dimension parameters are taken into account simultaneously. This applied optimization procedure protects the machine from full breakdown by adding a constraint on total rotor losses level. Since classical empirical analytical formula for losses in magnets

or iron are not reliable when several harmonics of currents are injected, it is then necessary to use FEM [18] for calculation of the total losses. However, despite of the evolution in the computer performances, direct optimization with FEM is still complex and time-costly. Surrogate model-assisted optimization approach consists of replacing FEM by a fast analytical model [19]. Two ways to employ the surrogate model are applied in this paper, RSM approach and an optimization technique - EGO algorithm. However due to the inaccuracy of the surrogate model, the solution found is not always enough accurate. The EGO algorithm, one of surrogate-assisted algorithms, has been used successfully in the field of electromagnetic design optimization [20][21]. It uses the FEM in conjunction with a progressively built surrogate model whose accuracy increases with the search for optimal design [22]. By this way, EGO benefits from both the rapidity of surrogate model and the accuracy of FEM.

In this paper, the work is structured in three parts. In the first part, the component and the FEM of the five-phase PM machine are introduced and the optimization problem is presented. The effect of the geometry and the control strategy are combined in a common goal in order to improve the performances of the drive. In the second part, the optimization tools used in this paper, the RSM strategy and the EGO algorithms, are introduced briefly. In the third part, the optimization process is divided into three parts in order to illustrate progressively the special characteristics of the five-phase PM machine in the flux weakening mode.

## COMPONENT AND MODEL

Despite the advances in computing power over the last decade, the expense of running analysis codes remains non-neglecting. Though, single evaluations of finite element analysis codes can take for instance from a few minutes up to hours, even days, following the desired type of simulation. Complex numerical models are often non-robust, the analysis and even the mesh generation of such models failing for different design configurations. Moreover numerical models such as FEM are confronted to numerical noise, which affects or alters the convergence of the optimization algorithm, especially in the case of gradient-based algorithms. In a word, direct integration of FEM within an optimal design process is difficult. On the contrary, the surrogate models, mostly interpolating models, are noise-free. The main purpose of the use of surrogate models within an optimization process consists, through a significant overall time reduction of the optimization process, in avoiding heavy simulations with long computation time. But due to the inaccuracy of the surrogate models, the optimal found solution is not exact, and more important it might be infeasible, violating the constraints. In order to benefit from both the accuracy offered by the FEM and the fast prediction of a surrogate model, different optimization strategies assisted by surrogate model are proposed [27]. In this paper, two surrogate assisted optimization strategies are used: RSM and EGO, depending on the complexity of optimization problems.

### Model

The global objective of a machine is to produce torque with good efficiency in a required speed range. In case of five-phase PM machine the electromagnetic torque can be computed by:

$$(1) \quad T_{em} = K(E_1 I_1 \cos \varphi_1 + E_3 I_3 \cos \varphi_3) / \Omega$$

with  $E_k$ : the  $k$ -harmonic of electromotive force;  $I_k$ : the  $k$ -harmonic of current;  $K$ : a constant linked to the number of phases ;  $\Omega$ : the speed;  $\varphi_k$ : phase between  $k$ -harmonic of respective electromotive force and current. Thus, if the currents are controlled, there are 4 degrees of freedom to define a torque  $(I_1, \varphi_1, I_3, \varphi_3)$ .

The study is based on a FEM [18], which allows computing various kinds of losses developing in the machine. The losses in the iron and in the magnet are directly obtained by the FEM. This computation is particularly important at high speed when the magnetic flux densities vary with high frequencies.

Fig 1 shows the studied global model with its different inputs and outputs, where it can be seen how certain outputs are deduced directly from the FEM while others are calculated using analytical equations. The first 4 inputs of the model represent the two supplying current harmonics of 1<sup>st</sup> and 3<sup>rd</sup> order. While two main dimensions of the geometrical model structure can be modified using the last two inputs.

The control methods of efficiency maximization in electrical machines consider classically only Joule losses. This is due to the necessity to have analytical equations in real time calculations [10] [11]. The problem is that, at high speed iron and magnet losses are no more negligible comparing to Joule losses. Moreover, certain new concentrated winding topologies generate high level of iron and magnet losses which makes the classical control methods much less efficient. Thanks to the FEM, a high speed optimum control strategy will be obtained looking for a compromise between the different losses with the respect of local constraints linked to the heating.

Generally, in common machine structures, the 3<sup>rd</sup> harmonic of the electromotive force is small comparing to the fundamental one. For this reason the classical method of efficiency maximization put the 3<sup>rd</sup> current harmonic equal to zero to avoid more Joule losses since this harmonic is not able to produce torque. This paper investigates the effect of the 3<sup>rd</sup> current harmonic on the global efficiency at high speed, taking into account not only circuits copper losses but also iron and magnet losses. Furthermore, this study is done

considering a limited DC bus voltage (where a phase voltage limitation is imposed) which allows checking the influence of the 3<sup>rd</sup> current harmonic on flux weakening operation.

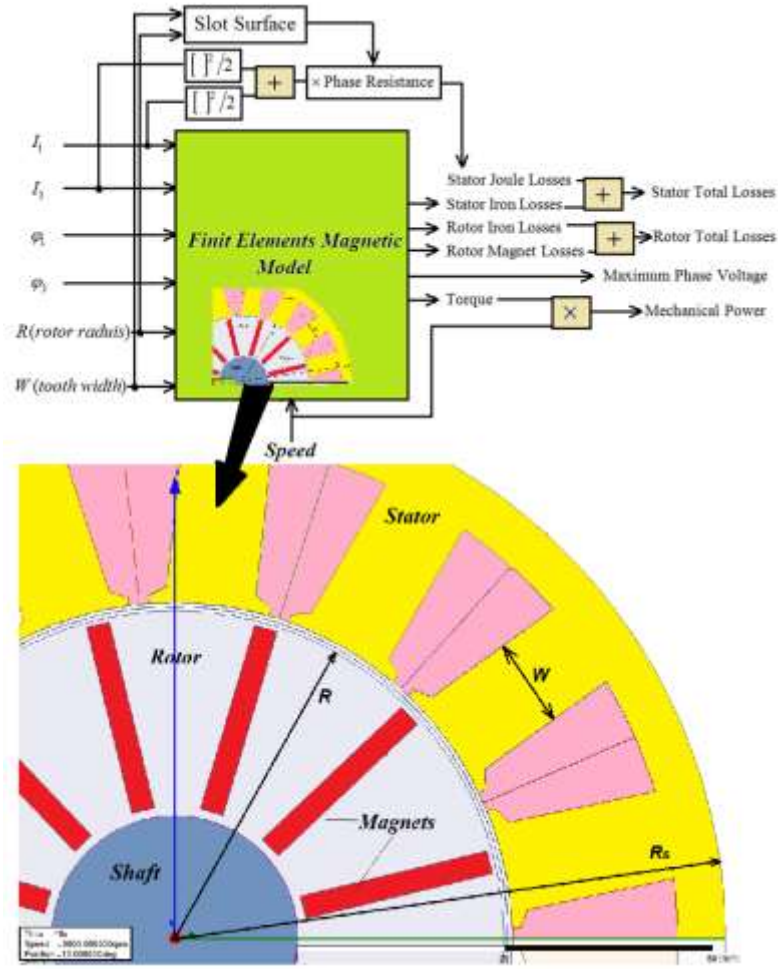


Fig. 1 Inputs and outputs of studied global model of the PM 5-phase machine

### Optimization problem

The objective of this paper is to minimize the total losses in order to improve the machine efficiency by taking into account the combination of the control variables and the geometrical ones. Five-phase structure adds a freedom degree to the control strategy of synchronous machine by allowing injecting the 3<sup>rd</sup> harmonic of current. This property increases the number of input parameters in flux weakening strategy from two, in the case of 3-phase machine (fundamental current amplitude and phase ( $I_1, \varphi_1$ )), to four in the case of 5-phase machine ( $I_1, \varphi_1, I_3, \varphi_3$ ) [1]. The added parameters can have a remarkable effect on iron and magnet losses in concentrated windings structure especially with the influence of iron nonlinearity. Additionally, two dimension variables are added in order to take into account the machine structure optimization: the rotor radius and the stator tooth width (tooth width + slot width = constant) (see Fig. 1). Both added parameters have remarkable effects on the objective function. The increase of the rotor radius decreases the height of the stator slots causing more copper losses (smaller copper section), and vice versa. The increase of the stator slot width (by decreasing the tooth width) expands the copper section leading to less copper losses but to higher iron saturation. Furthermore, the machine magnetic structure depends widely on the two optimized dimensions which gives these parameters an important influence on the machine torque and eddy-current losses. There are four inequality constraints and one equality constraint. The rotational speed of machine is fixed to 16 000 rpm which is the allowed maximal speed in flux-weakening zone. The motor power should be more than 10 kW. In order to avoid the demagnetization of the magnet, the rotor losses due to eddy currents in magnets and iron should be less than 400 W. The stator losses consisting in copper and iron losses should be less than 800W. The voltage per phase should be less than 70 V, due to the limit of the DC voltage bus supply.

The optimization problem is presented in Eqn. (2):

$$(2) \quad \min_{I_1, \varphi_1, I_3, \varphi_3, R, W} (Total Losses)$$

$$s.t. \quad Speed = 16000rpm, \quad Power \geq 10kW, \quad Losses_{rotor} \leq 400W, \quad Losses_{stator} \leq 800W, \quad \max(|U_{phase}|) \leq 70V$$

$$With \quad I_1 \in [0, 230](A), \quad \varphi_1 \in [-85, -60](^\circ), \quad I_3 \in [0, 25](A), \quad |\varphi_3| \in [0, 90](^\circ), \quad R \in [35, 60](mm), \quad W \in [3, 13](mm)$$

and Speed – maximum rotational speed, Power – power generated by the machine at maximum speed, Losses<sub>rotor</sub> – losses in rotor (iron + magnets), Losses<sub>stator</sub> – losses in stator (iron + windings), (U<sub>phase</sub>) – needed phase voltage, Total Losses – Losses<sub>stator</sub> + Losses<sub>rotor</sub>. The chosen range for the phase,  $\varphi_1$  and  $\varphi_3$  are in adequacy to the fact that the machine is working in flux-weakening mode,  $R$  – the radius of rotor,  $W$  – the stator tooth width (see fig.1 - the machine shape).

## OPTIMIZATION TOOLS

Two methodologies based on surrogate models are used to solve the constraint optimization problem. With the two methods, the iterative process is done with a human in the loop. Indeed, the complexity of the FEM and the optimization tools haven't been linked by programming allowing a study at each iteration.

### Response surface methodology (RSM)

To do quickly and easily an optimization process on a complex model, not necessary well linked with the optimization tool, a good way is to use a methodology based on response surfaces or also named surrogate model. The nature of the response surface can be multiple [23]. In this work, Kriging models are used for their good performances in the case of a low number of samples. A surrogate model must be built for the objective function (*Total Losses*), but also for each constraint of the optimization problem (*Power*, *Losses<sub>rotor</sub>*, *Losses<sub>stator</sub>*, *U<sub>phase</sub>*). The optimization process is performed through these surrogate models that are fast and thus allow reducing highly the computation time.

But, due to the inaccuracy of surrogate models, the solution found is not always enough accurate on the objective function or on the constraints. Therefore, it is wise to enhance the accuracy of the model using further function calls (infill or update points): new samples coming from fine model (FEM) are added. A method, more or less, sophisticated can be used to add, one or a set of new samples allowing to increase the accuracy [28]. However, the problem of finding the global optimum is not obvious with this technic if no exploration mechanism is applied. If the optimization problem is smooth, this technic is effective.

### Efficient Global Optimization (EGO)

The EGO algorithm is a surrogate-based optimization algorithm which uses Kriging models as surrogates for the fine model, in order to guide the search for the optimal solution. At each iteration of the algorithm, the improvement of solution is sought through an internal optimization loop, based on surrogate models. This optimization consists in the maximization of an Infill Criterion (IC) whose expression is based on the Kriging model prediction  $\hat{y}$  and an estimate of the prediction error  $\hat{s}$  [23]. The considered IC naturally balances the exploration of the design space, improving thus the quality of the Kriging surrogate model and the exploitation of promising regions of the design space in the search for improving solutions. By this way, the number of fine model (FEM) calls is drastically reduced, obtaining thus the optimal trade-off solutions with an affordable computational cost. The role of the surrogate model within the algorithm is to guide the search for improving solution.

The computational flow diagram of the EGO algorithm can be found in [21], and it is described in 8 steps as follows:

- Step 1). Initialization of the sampling plan: Select the initial designs of the sampling plan using Latin Hypercube strategy (generally a good choose for this kind of surrogate model).
- Step 2). Fine model evaluation: Evaluate the designs of the sampling plane with the fine model.
- Step 3). Kriging model construction: Build the Kriging models (see appendix) for each objective and constraint functions.
- Step 4). Improvement point search: Find the improvement point using the Infill Criterion (IC), expressed in equation (3).

$$(3) \quad \max_x \left[ E[I(x)] \cdot \prod P_{\exp}(x) \right]$$

$$\text{Subject to } g_{\text{inexp}}(x) \leq 0$$

Where  $E[I(x)]$  is the Expected Improvement (EI) which is the probability that the estimated response is smaller than the current minimal objective function;  $P_{\exp}(x)$  is the cumulative distribution function;

$g_{inexp}$  is the inexpensive constraint in terms of evaluation time. Details on the IC can be found in [23] and [29].

Step 5). Infill point fine model evaluation: Evaluate the infill point determined at the precedent iteration using the fine model (FEM).

Step 6). Best objective value: If the objective infill is lower than the best objective and constraint violation is in acceptable tolerance, set this point as the new best point.

Step 7). Sampled data addition: Add the infill point to the sampled data set.

Step 8). Stop criterion verification: If the maximum iteration number is attained, the algorithm ends. Otherwise, return to the step 3) and repeat.

The expected improvement ( $EI$ ) criterion was first used by Schonlau [22]. The  $EI$  criterion quantifies the amount of improvement expected to be attained by sampling at a certain point. The mathematical formulation of the  $EI$  criterion is given in (4).

$$(4) \quad EI = E[I(\mathbf{x})] = \begin{cases} (f_{min} - \hat{y})\Phi\left(\frac{f_{min} - \hat{y}}{\hat{s}}\right) + \hat{s}\phi\left(\frac{f_{min} - \hat{y}}{\hat{s}}\right) & \text{if } \hat{s} > 0 \\ 0 & \text{if } \hat{s} = 0 \end{cases}$$

where  $\phi$  and  $\Phi$  represent the normal probability density function, respectively the normal cumulative distribution function. Within the expression of  $EI$  we can distinguish the two terms corresponding to the *exploitation* of the surrogate models (first term), respectively the *exploration* of the design space (second term). When the value of the predicted error  $\hat{s}$  is zero (i.e. point already sampled), the  $EI$  becomes null, meaning that for this point there is no expectation of improvement. If the predicted error  $\hat{s}$  is different from zero, but small, and the predicted value of the function  $\hat{y}$  is very small, in compare to the current best known value of the function  $f_{min}$ , then the first term of the expression (4) becomes predominant. Though, the search is performed locally, exploiting the good accuracy of the surrogate models prediction. Otherwise, if the predicted error  $\hat{s}$  is important, then the second term in (4) takes control, looking to explore areas of the design space with high surrogate model inaccuracy.

Thus, the optimization's algorithm is applied not directly to the surrogate model but well to  $EI$ , which makes it possible to have two complementary mechanisms (exploitation / exploration) allowing a more robust convergence. The use of the surrogate model makes it possible to highly reduce the evaluation number of the fine model (here FEM to compute the losses).

## DESIGN PROCESS BASED ON OPTIMIZATION

Three flux weakening optimizations problems are gradually formulated according to the number of design variables. The first problem which is presented in Eqn. (5) takes into account the fundamental current. The second one which is presented in Eqn. (6) takes into account both the fundamental and the 3<sup>rd</sup> harmonic current according to the special characteristics of the five-phase machine. The optimization process starts with the selection of an initial sampling plan of 50 points. The initial sampling plan is then evaluated using the fine model and the objective and constraint function values are obtained. Next, for each objective and constraint function, a Kriging surrogate model is fitted over the initial sampling plan. A first optimization test is performed directly on both surrogate models. According to the optimization results for the two problems, two optimization strategies are used: exploration surrogate model strategy (EGO) and exploitation one (RSM – response surface methodology). The final optimization results of the two problems are compared. This approach shows clearly the advantage of control for multiphase machines to use all the available degrees of freedom. The third problem which takes into account not only the control variables (the fundamental and 3<sup>rd</sup> currents), but also the two dimension variables, is presented in Eqn. (2). The final comparison results between the three problems show the advantages of this optimization approach for the five-phase PM machine.

### Initial control solution

The optimization problem considers in this case is only with the fundamental currents. There are 2 design variables:  $(I_1, \varphi_1)$ . Eqn. (5) presents the optimization problem. This problem is the simplest one among the three optimization problems. The purpose of this step is to check the finite element model of the PM machine, it also allows finding the optimal solutions with constraints.

$$(5) \quad \min_{I_1, \varphi_1, I_3=0} (Total \text{ Losses})$$

$$s.t. \quad Speed = 16000rpm, Power \geq 10kW, Losses_{rotor} \leq 400W, Losses_{stator} \leq 800W, \max(|U_{phase}|) \leq 70V$$

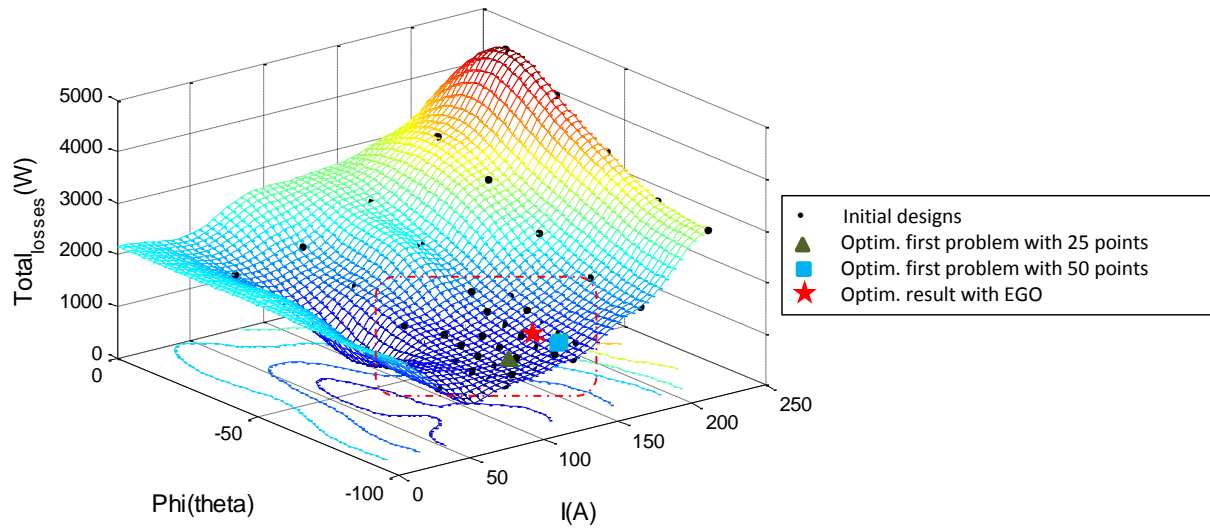
with  $I_1 \in [0, 230](A)$ ,  $\varphi_1 \in [-85, -60](^\circ)$

And the signification of the variables is the same with the Eqn. (1).

An initial set of 25 designs was chosen using the full factorial design. The set of designs were then evaluated in parallel on the available computer cores by the FEM. The Kriging models for each objective and constraint functions are built individually using the initial points. Fig. 1 presents the Response Surface (RS) of the total losses function for this optimization problem. The initial set of 25 designs is marked with the black dots. The optimal solution of this model (green triangle in Fig. 2) was sought using algorithm Sequential Quadratic Programming (SQP). The optimal solution was then validated using the FEM. According to the expert, the optimization result corresponds well the experiment. Another 25 points surrounded by the red dotted rectangle in Fig. 2 around the optimal one are selected and evaluated by the FEM. The new Kriging models for the objective and the constraints functions are then fitted with the 50 points. The optimal solution with the new models is presented in Fig. 2 by the blue square.

The model of the objective function presented in Fig.2 is complex. Therefore the exploration surrogate model strategy is employed. The EGO algorithm is then used on the Kriging model with 50 initial points. Instead of direct optimization with the surrogate model, the EGO algorithm maximizes the Expected Improvement (EI) in order to find the infill point which allows improving the model in the most incertitude zone. Once a point found, it is then evaluated with the FEM and added to the set of sampled data in order to build new Kriging models on the increased data set. The model accuracy increases progressively with the increase of the sample data. The algorithm stops when the stop criterion is satisfied, returning the final optimal solution which is validated by the FEM. Considering the time consuming FEM model, a total budget of 50 fine model evaluation is imposed. The final solution with EGO algorithm is marked by the red star.

With the set of 25 points, no solution was found with all the constraints. A modification of voltage constraint to 100V instead of 70V was then chosen and allows finding a solution  $(I_1, \varphi_1) = (114.16, -80.3)$  that verifies the constraints with acceptable tolerance (9886W for the power instead of 10 000W). After adding 25 points around the initial optimal solution, a new optimization with these 50 points is done with the voltage constraint of 70 V. An optimal solution is found with the respected constraints  $(I_1, \varphi_1) = (144.3, -82.2)$ .



**Figure 2.** Kriging model and Optimization results

The table 1 presents a comparison. For each optimal value  $(I_1, \varphi_1)$  found by Kriging model for a set of 25 or 50 points, the values of Power, Losses and Voltage are given using at first the Kriging model (square in grey) and secondly the FEM model (underlined). Relative errors are then provided in order to compare result obtained with Kriging model to those calculated with FEM, FEM results being taken as reference.

With the set of 25 points, it appears that the Kriging model leads to an error of more than 30% for the power and rotor losses. With the set of 50 points which allows verifying the voltage constraint of 70V, the error is weak for all variables except of the power (50.9%).

With EGO, the solution  $(I_1, \varphi_1) = (142.9, -75.9)$  is verifying all the constraints if a tolerance of 2.5V (less than 5%) is accepted for the voltage.

**Table 1.** Optimal solution with the first optimization problem

	$I_1$ (A)	$\varphi_1$ ( $^\circ$ )	Power (W)	$Losses_{rotor}$ (W)	$Losses_{stator}$ (W)	$U_{phase}$ (V)	Total losses (W)
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25 points	114.16	-80.3	9886	384.8	657.9	100	956.9
			<u>7456</u>	<u>288.6</u>	<u>641.6</u>	<u>109.6</u>	<u>930.2</u>
			32,6%	33,3%	2,5%	-8,8%	2,9%
50 points	144.30	-82.2	9886	394.9	783.3	70	1174.3
			<u>6551</u>	<u>376.3</u>	<u>798.7</u>	<u>71.1</u>	<u>1149.2</u>
			50,9%	4,9%	-1,9%	-1,5%	2,2%
Final solution (with EGO)	142.90	-75.9	<u>10020</u>	<u>379.7</u>	<u>798.7</u>	<u>72.5</u>	<u>1178.5</u>

Using the RSM, all the optimal found solutions calculated with FEM do not verify the constraints. However, a feasible solution can be found with EGO algorithm. Analysis of results of optimization process shows that the voltage constraint is the most pregnant. As consequence, it has been decided to explore the impact of injecting third harmonic currents in order to attenuate the pressure due to the DC bus voltage. In the following part, the optimization on a second problem will be presented. The same constraints and objective are present; nevertheless, there are four design variables instead of two ones.

### Control with the 3<sup>rd</sup> harmonic

Through the analysis between the initial solution and the one with the 3<sup>rd</sup> harmonic current, the surrogate model here is much smoother, and the voltage constraint here is easier to be verified. And finally, an optimization problem with the 3<sup>rd</sup> harmonic current is less constrained, less saturated and thus much easier. The RSM approach is sufficient allows finding the optimal results.

The optimization problem with 4 design variables is presented in Eqn. (6). The both optimization problems (1), (5) and (6) have the same objective and constraints.

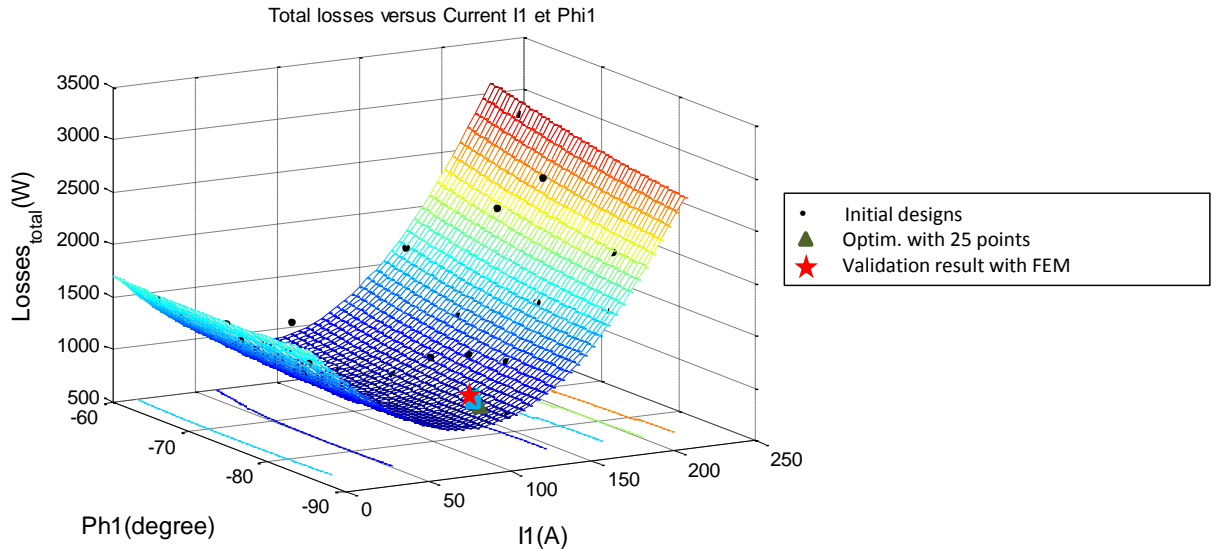
$$(6) \quad \min_{I_1, \varphi_1, I_3, \varphi_3} (Total\ Losses)$$

$$s.t. \quad Speed = 16000rpm, \quad Power \geq 10kW, \quad Losses_{rotor} \leq 400W, \quad Losses_{stator} \leq 800W, \quad \max(|U_{phase}|) \leq 70V$$

$$With \quad I_1 \in [0, 230](A), \quad \varphi_1 \in [-85, -60] (^{\circ}), \quad I_3 \in [0, 25] (A), \quad |\varphi_3| \in [0, 90] (^{\circ})$$

And the signification of the variables is the same with the Eqn. (1).

As in the first optimization problem, a first set of 25 points is selected. The Kriging model of the objective function with the 25 initial designs (black points) is presented in Fig. 3. As we can see that the Kriging model with four design variables is less complicated than the previous one with two design variables. The first optimal solution of this model (green triangle in Fig. 3) is sought using algorithm Sequential Quadratic Programming (SQP) with multi-start strategy. The solution validated by FEM is marked with a red filled star. The both solutions (Kriging model and FEM) are very close, and the Kriging model can be considered sufficiently accurate. The exploitation surrogate model optimization strategy is hence chosen for this problem. It means that the infill points at the optimum predicted by the surrogate model will be progressively added to the sampling plan.





**Figure 3.** Kriging model with 25 samples and Optimization results with 4 design variables

The table 2 presents the improvement process of optimization by iteration. The comparison between the optimal solutions and the FEM evaluation result underlined at the optimum is presented respectively in the table 2. All the optimal solutions respect the constraints, but the FEM results are not satisfied until the one with 45 points. The first line presents the results with 25 points, and both the torque and the voltage constraints are not respected. After adding 10 points to the sampling plane, only the voltage constraint (less than 70V) of the FEM evaluation is not respected.

**Table 2. Optimal solution with the second optimization problem**

	$I_1$ (A)	$\varphi_1$ (°)	$I_3$ (A)	$\varphi_3$ (°)	Power (W)	Losses <sub>rotor</sub> (W)	Losses <sub>stator</sub> (W)	$U_{phase}$ (V)	Total losses (W)
25pts	126.7	-78.4	15.02	30	9886	202.0	643.6	70	845.6
					<u>9684</u>	<u>201.2</u>	<u>677.9</u>	<u>72.5</u>	<u>879.0</u>
					2,1%	0,4%	-5,1%	-3,4%	-3,8%
35pts	128.4	-78.4	13.93	25.7	9886	207.9	648.6	70	870.3
					10941	<u>209.5</u>	<u>662.6</u>	<u>72.1</u>	<u>872.2</u>
					-9,6%	-0,8%	-2,1%	-2,9%	-0,2%
45pts (final)	129.2	-79.1	13.48	19.5	9953	213.3	666.4	69.93	879.6
					<u>9953</u>	<u>213.3</u>	<u>666.5</u>	<u>69.91</u>	<u>879.8</u>

The exploitation surrogate model optimization allows finding the feasible solution for the 4 design variables.

The two optimization problems are compared in this part. The table 3 presents the comparison between the optimal solutions. By injecting the 3<sup>rd</sup> harmonic currents, the voltage constraint is respected while the mechanical torque is kept. Furthermore, the total losses in the machine decrease 25%. The comparison can well illustrate the advantages for 5-phase machines to inject third-harmonic component.

**Table 3. Comparison between the first two optimization problems**

	$I_1$ (A)	$\varphi_1$ (°)	$I_3$ (A)	$\varphi_3$ (°)	Torque (Nm)	Power (W)	Losses <sub>rotor</sub> (W)	Losses <sub>stator</sub> (W)	$U_{phase}$ (V)	Total losses (W)
1 <sup>st</sup> problem	142.9	-75.9	0	0	6.0	10020	379.7	798.7	<u>72.5</u>	<u>1178.5</u>
2 <sup>nd</sup> problem	129.2	-79.1	13.5	19.5	5.9	9953	213.3	666.5	69.9	<u>879.8</u>

Two optimization strategies are employed respectively for the two problems: exploration surrogate model optimization and exploitation one. The choice of the most appropriate optimization strategy depends on the model complexity. If the model to be approximated is smooth and not complex, the exploitation strategy (RS) can be employed; otherwise the exploration one (EGO) should be used.

#### Adding geometrical degree of freedom

The flux weakening control strategy is accomplished in the previous part. In this part, the shape design optimization is presented.

The objective of this part is to optimize design of the 5-phase high speed machine. Compared to the 4 design variables optimization problem, two dimension variables are added in order to take into account the machine structure optimization: the rotor radius and the stator tooth width (tooth width + slot width = constant) (see Fig. 1). The same objective and constraints are considered compared with the two previous problems. The optimization problem is presented in Eqn. (2).

As the number of design variables increases, it is difficult to have an accurate surrogate model. There are two approaches to enhance the accuracy of surrogate model: increase the sampling points and use the appropriate sampling strategy. In our case, an initial set of 70 points using Latin Hypercube strategy is selected for the 6 design variable problem. The EGO algorithm is used in order to obtain a global optimum and have an accurate surrogate model around the optimum. A total budget of 200 fine model evaluations is imposed during the EGO optimization process.

The table 4 presents the comparison results between the 4 and 6 design variable problems. The initial dimension parameters are considered for problem with four variables.

**Table 4. Optimal solution comparison**

	$I_l$ (A)	$\varphi_l$ (°)	$I_3$ (A)	$\varphi_3$ (°)	$R$ (mm)	$W$ (mm)	Power (W)	$Losses_{rotor}$ (W)	$Losses_{stator}$ (W)	$U_{phase}$ (V)	Total Losses (W)
4 variables	129.2	-79.1	13.5	19.5	45.0	7.0	9953	213.3	666.5	69.9	<u>879.8</u>
6 variables	159.4	-76.2	5.2	71.4	43.0	4.2	10640	163.5	675.8	63.8	<u>839.3</u>

After adding two dimension parameters, the high speed machine can improve notably the performance at the optimal solution. The critical rotor losses are reduced (23%) while all the constraints are respected. Moreover the final optimal solution can work with lower DC voltage bus supply (-9%) and higher mechanical power (+7%).

This work is done in the condition of a point of high speed operation. If the PM machine should be designed with variable speeds, it would be necessary to reformulate the optimization problem, and that is another job to complete this one.

## CONCLUSIONS

Multi-phase machines are widely used in automotive sector for the reasons as reliability, smooth torque and partition of power. Among the different kinds of multi-phase machines, the synchronous PM one appears as an attractive solution because of the high ratio of torque/volume. The five-phase PM machines can add a degree of freedom which increases in certain cases the flexibility of the control. Whereas, by injecting relatively low 3<sup>rd</sup> harmonic of current (~10% of fundamental), the DC bus voltage constraint on the PWM VSI are easier to be respected. Moreover the higher harmonic current influences on the rotor losses which are non-negligible can be taken into account, and thus the total losses. The optimization results have proved the remarkable effect of using the freedom degree offered by a 5-phase structure on iron and magnets losses, and the total losses are notably reduced (25%). Moreover, due to this optimization procedure rotor losses are decreased far below the imposed limit (47%), which increases the machine protection against magnet demagnetization. The RSM and EGO algorithm can be employed respectively depending on the complexity of the surrogate model. The RSM approach is more effective for a simple problem where the surrogate model is smooth. Nevertheless, the EGO algorithm is preferred to solve a more complex problem, which will allow obtaining progressively the global optimal solution of the FEM with small evaluation budget. Combining with two geometric parameters, a more complex optimization control problem is formulated and resolved in order to explore the sensitivity of the result in relation to geometric parameters. The performances of the 5-phase machine with concentrated windings are notably improved at high speed (16 000 rpm).

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## APPENDIX

### Basis of the Kriging method

Kriging method was first developed by D. Krige and was introduced in field of computer science and engineering by Sacks *et Al* [10]. In Kriging model, an unknown function  $y$  can be expressed as in (7):

$$(7) \quad y = B(x) + Z(x)$$

where  $B(x)$  is a regression or polynomial model, giving the global trend of the modeled function  $y$ , and  $Z(x)$ , which is a model of stochastic process, gives the local deviations from the global trend. The Gaussian correlation function is chosen in order to control the smoothness of the model.

The mean square error (MSE) is the expected value of difference between the true response and the estimated one. By minimizing the expected MSE, the expression for the Kriging model is:

$$(8) \quad \hat{y}(x) = \mathbf{f}\hat{B} + r^T R^{-1} (y - \mathbf{f}\hat{B})$$

where  $\mathbf{f}$  is a unit vector with length equal to the number of sampled points,  $\hat{B}$  is the estimator for the regression model,  $r$  is a correlation vector between a new location  $x$  to be estimated and the sample points location,  $y$  is the true response vector of the sampled points.