YIC2013
Second ECCOMAS Young Investigators Conference
2–6 September 2013, Bordeaux, France

A reduced numerical strategy based on PGD for composite shell structures simulations

E. Prulière [a,*], S. Metoui [a],

[a] Intitut de mécanique et d'ingénierie de Bordeaux
Arts et Métiers ParisTech, Esplanade des Arts et Métiers, 33400 Talence, France

*e.pruliere@i2m.u-bordeaux1.fr

Abstract. *This paper explores an alternative to shell computation. The proposed strategy uses the Proper Generalized Methods based on a separated representation. The idea is to solve the full 3D solid problem separating the in-plane and the out-of-plane spaces. This allows to represents complex fields in the thickness without the complexity and the computational cost of a solid mesh which is particularly interesting when dealing with multi-layer composite.*

Keywords: Shell structures; Composites; Proper Generalized Decomposition.

---

1 INTRODUCTION

The composite structures are more and more used because of the need in structures lightening especially in aeronautical applications. The damaging process in composite structures is of great complexity especially under dynamic solicitations. Organic matrix composites structures are generally composed of three characteristic scales. The microscopic scale is related to the fibres arrangement in the matrix, the mesoscopic scale is related to the plies and the macroscopic scale is related to the structures. Damage can occur at these three scales:

- fibres ruptures, fibres/matrix decohesion and matrix cracking at microscopic scale,
- delamination at mesoscopic scale,
- macroscopic ruptures.

Thus, the damaging of composites is a multidimensional phenomenon. An efficient model must be able to consider the different damaging types and their scales and the interaction between them. Another constrain to a good prediction of damages is that a good model must be able to be solved with numerical solver. With the classical numerical tools, it is definitely not conceivable to perform a complete multiscale simulation considering fibres and structures.

2 NUMERICAL STRATEGIES

2.1 Restriction of classic numerical strategies

The mainly used numerical method for mechanic simulations is the finite element method (FEM). For example, numerical simulations of composite structures are generally performed using multi-layered shell elements in the context of the finite elements method. This strategy has numerous advantages like a low computation time and the capability to reproduce the comportment of composites in most of cases. The main restriction of this approach is that it has only a coarse description of strain and stresses variations in the thickness. This approximation is no more valid when increasing the thickness, near the boundary and loading conditions and when non linear phenomena like delamination or other damaging processes occurs in the thickness. Consequently other strategies must be developed for damaging simulations.

2.2 The Proper Generalized Decomposition

Model reduction techniques are being developed in computational mechanics. This kind of strategies has prove its efficacy in many applications like thermal problems [3], stochastic simulations, rheology ([1], [2], [4]) and structural mechanics [5]. This techniques are based on a separated representation of the solution. Methods based on the proper orthognal decomposition (POD) needs a precomputation to build a reduced approximation basis and to find the solution. So it is needed to know the solution prior to compute the approximation basis. Others strategies likes the A priori Hyper Reduction Method, allows building the spacial approximation basis and the solution at the same time. Another method consist in searching the solution directly on a defined separated form. This method is called the Proper Generalized Decomposition (PGD). Details on the method can be found here [?]. A resolution with the PGD needs:

1. An equation to solve, i.e. a partial differential equation modelling the physical phenomena.
2. A separated decomposition of the unknown in order to build the solution.
3. An approximation of the solution on each subspace (a Finite Element approximation is often used).
4. Some boundary conditions defined on the subspaces (the application of boundary conditions may be a little complicated)

3 USING THE PGD ON SHELL STRUCTURES

3.1 Mechanical model

The model used is based on the classical momentum conservation equation:

$$\mathrm{div}\boldsymbol{\sigma} + \mathbf{f} = \rho\boldsymbol{\Gamma} \tag{1}$$

where $\boldsymbol{\sigma}$ denotes the stress tensor, div denotes the tensorial divergence,$\mathbf{f}$ is the volume force, ρ is the density and $\boldsymbol{\Gamma}$ is the acceleration. $\boldsymbol{\sigma}$ is linked to the deformation by the constitutive relation:

$$\boldsymbol{\sigma} = \mathbf{H}\boldsymbol{\varepsilon} \tag{2}$$

where $\boldsymbol{\varepsilon}$ is the strain tensor and $\mathbf{H}$ is the fourth order rigidity tensor.
$\boldsymbol{\varepsilon}$ is the symmetric gradient of the displacement $\mathbf{u} = (u, v, w)$.
The weak formulation of the equilibrium equation Eq. (**??**) without dynamic effect is:

$$\int_\Omega \varepsilon(\mathbf{u}^\star) : (\mathbf{H}\varepsilon(\mathbf{u})) = \int_\Omega \mathbf{u}^\star.\mathbf{f} + \int_{\partial\Omega} \mathbf{u}^\star.(\sigma.\mathbf{n}) \tag{3}$$

where Ω is the domain taken by the structure.

3.2 Separated representation

The idea is to use the PGD method separating a solid shell structure in two spaces: the mid-plane surface and the thickness. The shell structure occupies a region $\Omega = \mathcal{S} \times \mathcal{T}$ where $\mathcal{S}$ describes the mid-plane surface and $\mathcal{T} = [-\frac{e}{2}; \frac{e}{2}]$ is an interval containing all the positions in the thickness defined by a signed distance from the mid-plane. e denotes the maximum thickness of the shell.

$$\left\{ \begin{aligned} u(\mathbf{X}, z) &\approx \sum_{i=1}^{N} F_i^u(\mathbf{X}) G_i^u(z) \\ v(\mathbf{X}, z) &\approx \sum_{i=1}^{N} F_i^v(\mathbf{X}) G_i^v(z) \quad \forall \mathbf{X} \in \mathcal{S} \\ w(\mathbf{X}, z) &\approx \sum_{i=1}^{N} F_i^w(\mathbf{X}) G_i^w(z) \end{aligned} \right. \tag{4}$$

$\mathbf{X}$ is the position vector and z is the local out-of-plane coordinate. The vector function $\mathbf{F}_i = (F_i^u, F_i^v, F_i^w)$ is defined over the mid-plane surface $\mathcal{S}$ and the vector function $\mathbf{G}_i = (G_i^u, G_i^v, G_i^w)$ is defined over the thickness of the shell structure.

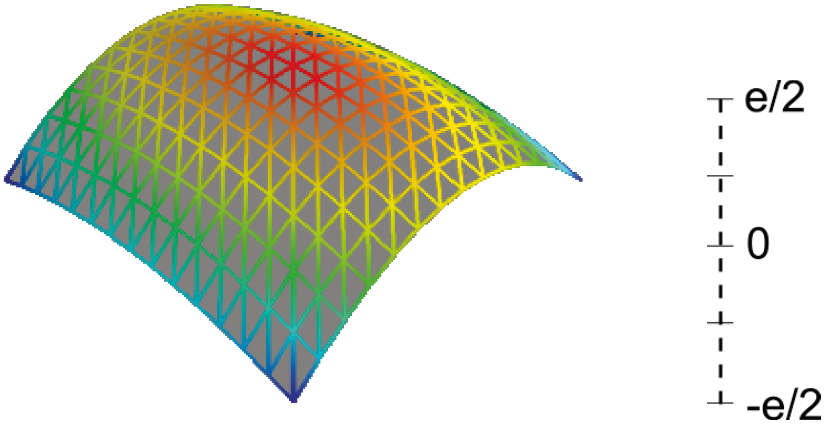

Figure 1: Example of a triangular shell mesh for the mid plane surface (left) and a 1D mesh for the thickness (right).

3.3 Approximation on the subspace

The vector functions $\mathbf{F}_i$ are approximated using shell or a plate finite elements. The shell mesh describe the position on the mid-plane. The $\mathbf{G}_i$ functions are approximated by simple 1D finite elements describing the position on the thickness. It can be noticed that a 3D structured mesh of the shell structure can be derived from the two previous meshes (depicted in Figure 1).

3.4 References format

The bibliography should be the last section of the paper and use the layout as given in this example and 9pt font. Use an unnumbered heading 'References' in the primary heading format. For a book reference use example [**?**]. For a journal article use [**?**]. An article in proceedings should look like [**?**]. The references should be labeled in the order in which they appear in the text. Place references in the text using a number or a list of numbers between square brackets.

4 CONCLUSIONS

In this brief article, a promising numerical strategy has been exposed. This strategy permits a full 3D modeling of structures with the computational complexity of a non linear shell modelling. This approach is interesting in order to simulate composites damaging with a low computational coast.

REFERENCES

[1] A. Ammar, B. Mokdad, F. Chinesta, R. Keunings. A New Family of Solvers for Some Classes of Multidimensional Partial Differential Equations Encountered in Kinetic Theory Modeling of Complex Fluids. *J. Non-Newtonian Fluid Mech.* **139**, 153-176, 2006.

[2] A. Ammar, B. Mokdad, F. Chinesta, R. Keunings. A New Family of Solvers for Some Classes of Multidimensional Partial Differential Equations Encountered in Kinetic Theory Modeling of Complex Fluids. Part II: Transient Simulation Using Space-Time Separated Representation. *J. Non-Newtonian Fluid Mech.* **144**, 98-121, 2007.

[3] F. Chinesta, A. Ammar, F. Lemarchand, P. Beauchene, F. Boust, Alleviating mesh constraints: Model reduction, parallel time integration and high resolution homogenization, *Comput. Methods Appl. Mech. Engrg.* **197**, 400-413, 2008.

[4] E. Prulière, A. Ammar, N. El Kissi, F. Chinesta. Recirculating flows involving short fiber suspensions: numerical difficulties and efficient micro-macro solvers. *Archives of Computational Methods in Engineering*, **16**:1, 1-30, 2009.

[5] E. Prulire, J. Frec, F. Chinesta, A. Ammar. An efficient reduced simulation of residual stresses in composite forming processes. *Int. J. of Material Forming*, **3**:1, 2010.