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Tolerance analysis approach based on the classification of uncertainty (aleatory / epistemic)

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Abstract

Uncertainty is ubiquitous in tolerance analysis problem. This paper deals with tolerance analysis formulation, more particularly, with the uncertainty which is necessary to take into account into the foundation of this formulation. It presents:

- a brief view of the uncertainty classification: Aleatory uncertainty comes from the inherent uncertain nature and phenomena, and epistemic uncertainty comes from the lack of knowledge,
- a formulation of the tolerance analysis problem based on this classification,
- its development: Aleatory uncertainty is modeled by probability distributions while epistemic uncertainty is modeled by intervals; Monte Carlo simulation is employed for probabilistic analysis while nonlinear optimization is used for interval analysis.

1. Introduction

UNCERTAINTY is ubiquitous in any engineering system at any stage of product development and throughout a product life cycle. Examples of uncertainty are manufacturing imprecision, usage variations and manufactured geometric dimensions, which are all subjected to incomplete information. Such uncertainty has a significant impact on product performance. The product performance improvement with several uncertainty types is very important to avoid warranty returns and scraps.

Due to the imprecision associated with manufacturing process; it is not possible to attain the theoretical dimensions in a repetitive manner. That causes a variation of the product performance. In order to ensure the desired behavior and the performance of the engineering system in spite of uncertainty, the component features are assigned a tolerance zone within which the value of the feature i.e. situation and intrinsic properties. To manage the rate of out-of-tolerance products and to evaluate the impact of component tolerances on product performance, designers need to simulate the influences of uncertainty with respect to the functional requirements.

One of the most controversial discussions in uncertainty analysis relates to the classification of uncertainty into several types and the possible sources from where it emanates. A classical classification is the separation of uncertainty into the two types: aleatory and epistemic [1], [2], [3]. Aleatory uncertainty, also referred to as irreducible, objective or stochastic uncertainty, describes the intrinsic variability associated with a physical system or environment. According to the probability theory, aleatory uncertainty is modeled by random variables or stochastic processes. Epistemic uncertainty, on the other hand, is due to an incomplete knowledge about a physical system or environment. The definition and the classification of uncertainty are discussed in the section 2.

Based on this classification, a formulation of the tolerance analysis problem is proposed in the section 3. In fact, the component deviations are aleatory and so irreducible (due to manufacturing imprecision, aleatory uncertainty exists in the geometrical component dimensions); and the gaps between components are epistemic uncertainty (due to the complexity of system behavior with gaps, epistemic uncertainty exists in the behavior model; moreover, the worst gap configurations of the over-constrained system depend on the component deviations).

There is a strong need for tolerance analysis (uncertainty propagation) to estimate the probability expressed in ppm (defected product per million) with high-precision. Much effort has been spent on exploring the effect of aleatory uncertainty on systems, while very few investigations have been reported in studying epistemic uncertainty and the mixture of aleatory and epistemic uncertainties. Aleatory and epistemic uncertainty types exist simultaneously in real-world systems. Therefore, the main scientific challenge concerns the development of hybrid approaches mixing evidence and probability theories to propagate aleatory and epistemic uncertainty types for tolerance analysis. In the section 4, the first proposal adopts the following approaches to address this challenge: a mathematical formalization and its implementation based on coupled optimization and Monte Carlo Simulation.

2. Uncertainty

«The concept of uncertainty has starting with Socrates and Platon, philosophers doubted whether scientific knowledge, no matter how elaborate, sufficiently reflected reality (Kant, 1783). They realized that the more we gain insight into the mysteries of nature, the more we become aware of the limits of our knowledge about how 'things as such' are (Kant, 1783).» [4]

The concept of uncertainty is old. The term 'uncertainty' has come to encompass a multiplicity of concepts. A fundamental definition of uncertainty is "*liability to chance or accident*", "*doubtfulness or vagueness*", "*want of assurance or confidence; hesitation, irresolution*", and "*something not definitely known or knowable*" [5].

A significant amount of research has been devoted to the definition and classification of the uncertainty. A classical classification is the separation of uncertainty into the two types: aleatory and epistemic. Aleatory uncertainty is defined as the randomness or inherent variability of the nature, and it is objective and irreducible. Aleatory uncertainty is usually modeled by probability theory. Examples of this category include the dimensions of manufacturing parts and material properties. On the other hand, epistemic uncertainty is due to the lack of knowledge or the incompleteness of information. It is subjective and reducible. The assumptions made in building models are one example of epistemic uncertainty. Although intensive research has been conducted on aleatory uncertainty, few studies on epistemic uncertainty have been reported.

Study on epistemic uncertainty due to the lack of knowledge has received increasing attention in risk assessment, reliability analysis, decision-making, and design optimization. Epistemic uncertainty is sometimes referred to as state of knowledge uncertainty, subjective uncertainty, or reducible uncertainty, meaning that the uncertainty can be reduced through increased understanding (research), or increased and more relevant data. Epistemic quantities are sometimes referred to as quantities which have a fixed value in an analysis, but we do not know that fixed value. For example, the elastic modulus for the material in a specific component is presumably fixed but unknown or poorly known. This last point of view limits the definition of epistemic uncertainty to the parameter uncertainty. Moreover, some studies define epistemic uncertainty as the scientific uncertainty in the model. It is due to limited data and knowledge. Epistemic uncertainty consists not only of imprecision in parameter estimates, but also incompleteness in modelling, vagueness in appropriate engineering estimates, indefiniteness in the applicability of the model, and doubtfulness and vagueness in the interpretability of results produced by a model.

The idea of distinguishing between aleatory uncertainty and epistemic uncertainty sounds simple. In practice, the distinction between aleatory uncertainty and epistemic uncertainty can get confusing. An aleatory uncertainty is associated to parameter; therefore an epistemic uncertainty is usually restricted to epistemic parameter uncertainty.

In fact, some studies on Probability Risk Assessment propose to split the epistemic uncertainty into three categories: parameter, model, and completeness uncertainty.

"*Parameter uncertainties include not only imprecisions due to small samples of recorded data, but also uncertainties in experts' judgments of parameter values when there are not recorded data"*

"Model uncertainty can be divided into two subcategories: (1) Indefiniteness in the model's comprehensiveness (i.e., does the model account for all the variables which can significantly affect the results), (2) Indefiniteness in the model's characterization (i.e.,

refers to the uncertainties in the relations and descriptions used in the model. Even if the pertinent variables are included in the model, appropriate relationships among the variables may not be described"

"Completeness uncertainties are the uncertainties as to whether all the significant phenomena and all the significant relationships have been considered in the PRA (Probabilistic Risk Analysis). Completeness uncertainties are similar in nature to modelling uncertainties but occur at the initial stage in the PRA. There are two subcategories of completeness uncertainties: (1) Contributor uncertainties (i.e., uncertainty as to whether all the pertinent risks and all the important accidents have been included) and (2) Relationship uncertainties (i.e., uncertainty as to whether all the significant relationships are identified which exist among the contributors and variables)" [6]

In the following, we consider the scope of these definitions for the problem formalization.

3. Formulation of tolerance analysis problem with uncertainty point of view.

This section presents the formulation proposed by Dantan et al. [7], [8], [9] which has been adopted for tolerance analysis problem. This is followed by the identification of uncertainty associated to this formulation.

3.1. Geometrical model & Product behavior model

Tolerance analysis has become an important issue in product design process; it has to simulate the "real-world" of the product with the minimum of uncertainty. Tolerance analysis concerns the verification of the value of functional requirements after tolerance has been specified on each component. To do so, it is necessary to simulate the influences of component deviations on the geometrical behavior and the functional characteristics of the mechanism. The geometrical behavior model needs to be aware of the surface deviations of each component (situation deviations and intrinsic deviations) and relative displacements between components according to the gap. The approach used in this paper is a parameterization of deviations from theoretic geometry, the real geometry of parts is apprehended by a variation of the nominal geometry.

The deviation of component surfaces, the gaps between components and the functional characteristics are described by parameters:

- $X=\{x_1, x_2, \ldots., x_n\}$ are the parameters which represent each deviation (such as situation deviations or/and intrinsic deviations) of the components making up the mechanism.
- $G=\{g_1, g_2, \ldots., g_m\}$ are the parameters which represent each gap between components

The mathematical formulation of tolerance analysis takes into account the influence of geometrical deviations on the geometrical behavior of the mechanism and on the geometrical product requirements; all these physical phenomena are modeled by constraints on the parameters:

$C_c(X,G)= 0$

Composition relations of displacements in the various topological loops express the geometrical behavior of the mechanism. They define compatibility equations between the deviations and the gaps. The set of compatibility equations, obtained by the application of composition relation to the various cycles, makes a system of linear equations. So that the system of linear equations admits a solution, it is necessary that compatibility equations are checked.

$C_i(X,G) \leqslant 0$ and $C_{i*}(X,G) = 0$

Interface constraints limit the geometrical behavior of the mechanism and characterize non-interference or association between substitute surfaces, which are nominally in contact. These interface constraints limit the gaps between substitute surfaces. In the case of floating contact, the relative positions of substitute surfaces are constrained technologically by the non-interference, the interface constraints result in inequations. In the case of slipping and fixed contact, the relative positions of substitute surfaces are constrained technologically in a given configuration by a mechanical action. An association models this type of contact; the interface constraints result in equations.

$C_f(X,G) \leqslant 0$

The functional requirement limits the orientation and the location between surfaces, which are in functional relation. This requirement is a condition on the relative displacements between these surfaces. This condition could be expressed by constraints, which are inequations.

3.2. Uncertainty point of view

This formulation discussed in the previous paragraph is affected by uncertainties. Based on the classification of the uncertainty, we identify the associated uncertainty of the formulation (Figure 1). It includes aleatory uncertainty which is the manufacturing deviation of each component. Due to the imprecision associated with manufacturing process; it is not possible to manufacture any dimension to the exact theoretical value. Therefore a manufacturing deviation is an irreducible uncertainty. Aleatory uncertainty is modeled by random variables or stochastic processes by probability theory if information is sufficient to estimate probability distributions. Therefore, each component of X is continuous random variable.

The accuracy of a mathematical model to describe an actual physical system of interest depends on the model uncertainty. Model uncertainty, also known as model-form, structural, or prediction-error uncertainty, is a form of epistemic uncertainty. All models are unavoidable simplifications of the reality which leads to the less than ideal situation: every model is lacking to a certain degree the conditions of reality. In fact, the geometrical model does not usually take into account the form deviations and their impacts on the behavior model. This aspect is not covered in this paper.

A mechanism is a set of components in a given configuration with each components having deviations and the gaps that result through the given assembly configuration of components. These gaps induce displacements between parts. A configuration is a particular relative position of parts of an assembly depending of gaps without interference between parts. As the mechanism includes gaps, the relative location of functional surfaces depends on the configuration, which is not single. For the tolerance analysis, we don't know the configuration of gaps. We can consider a gap as parameter uncertainty, completeness uncertainty or free variable that is controversial discussion.

Mechanism can be divided into two main categories in terms of degree of freedom, Iso-constrained mechanisms, and over-constrained mechanisms. Given their impact on the response function formulation for the problem of tolerance analysis, a brief discussion of these two types is given by *Ballu et al.[10]*. Usually, tolerance analysis uses a relationship of the form: Y=f(X) where Y is the response (characteristic such as gap or functional characteristics) of the mechanism and the function f is the mechanism response function which represents the deviation accumulation. It could be an explicit analytic expression or an implicit analytic expression. For iso-constrained mechanism and simple over-constrained mechanism, it is possible to determine the worst configurations of gaps; therefore the function f is an explicit function. In this case, the epistemic uncertainty is reduced.

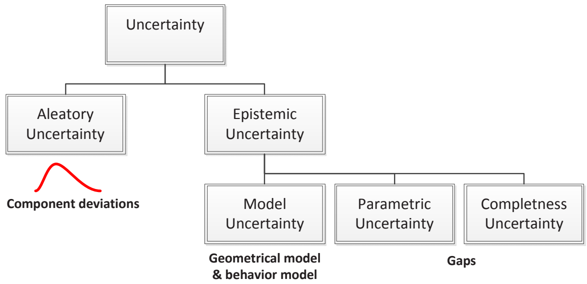

Fig. 1. Taxonomy of uncertainty

3.3. Formulation of tolerance analysis problem

This work generalizes and extends the earlier research carried out in the field of tolerance analysis by *Dantan et al.* [7], [8], [9] Using the mathematical existential and universal quantifiers, they simulate the influences of geometrical deviations on the geometrical behavior of the mechanism. Their approach translates the concept that a requirement must be respected in at least one acceptable configuration of gaps (existential quantifier there exists), or that a requirement must be respected in all acceptable configurations of gaps (universal quantifier for all). In order to formalize the problem, we proceed by adopting the semantics of the quantifier and the classification of the uncertainty. This is a two-step process consisting of evaluating the assemblability of the mechanism and respect of the functional conditions:

- The condition of the assemblability describes the essential condition for the existence of gaps that ensure the assembly of the components in the presence of the part deviations. In order for a mechanism to assemble successfully, the different components in the presence of deviations should assemble without interference and should have a specific set of gaps that characterize the instance of the assembly. This condition stipulates the use of an existential quantifier for an initial search for the existence of a feasible configuration of gaps: *"there exists an admissible gap configuration of the mechanism such that the assembly requirement (interface constraints) and the compatibility equations are respected" (Assemblability condition).*
- Once a mechanism assembles, in order to evaluate its performance under the influence of the deviations, it is necessary to describe an additional condition that evaluates its core functioning with respect to the basic product requirements. In terms of the tolerance analysis, the basic requirement becomes the maximum or minimum clearance on a required feature that would have an impact on the mechanism's performance. The most essential condition

therefore becomes that for all the possible gap configurations of the given set of components that assemble together, the functional condition imposed must be respected. In terms of quantification needs, in order to represent all possible gap configurations, the universal quantifier is required: *"for all admissible gap configurations of the mechanism, the geometrical behavior and the functional requirement are respected" (functional condition).*

Based on the formulation of the two conditions, we can add the uncertainty point of view:

- The respect of these two conditions (assemblability and functional requirement) is impacted by the aleatory uncertainty: the component deviations.
- The respect of the assemblability condition is facilitated by the gaps: the assemblability condition is respected if there exists one at least value of the epistemic uncertainty such that the compatibility equations and the interface constraints are respected.
- The respect of the functional condition is not facilitated by the gaps: the functional condition is respected if for all acceptable values of the epistemic uncertainty, all constraints (compatibility equations, interface constraints and functional requirements) are respected

Due to the aleatory uncertainty, this proposal focuses on statistical tolerance analysis. And, to improve the tolerancing process in an industrial context, there exists a strong need for statistical tolerance analysis to estimate the probability expressed in ppm (defected product per million) with high-precision computed at lower cost.

Let P_A be the probability of the assemblability for a given tolerance specification. This specifies the respect of the assemblability condition. Let P_{FR} be the probability of respect of the functional requirements. Let AC be the event that the assemblability condition for a given assembly are respected. Let FC be the event that the functional condition are fulfilled.

The probability expression of the two conditions can be translated as:

$$P_A = P(AC) = P(C_c(X,G) = 0 \cap C_i(X,G) \leqslant 0 \cap C_{i^*}(X,G) = 0) \quad (1)$$

G is considered as free parameters

$$P_{FR} = P(FC) = P(C_f(X,G) \leqslant 0,\ \forall G \in \{G \in \boldsymbol{R}^m : C_c(X,G) = 0 \cap C_i(X,G) \leqslant 0 \cap C_{i^*}(X,G) = 0\}) \quad (2)$$

The main scientific challenge concerns the development of approaches to propagate aleatory and epistemic uncertainties for tolerance analysis (aleatory uncertainties = component deviations; epistemic uncertainties = gap configurations). In addition to this, the second challenge is to evaluate the probability computation in an acceptable computing time and managing the accuracy of the results. It should be an area for some intense research on heterogeneous uncertainty propagation.

4. Development of the formulation

The purpose of this section concerns the development of a framework mixing evidence and probability theories to propagate aleatory and epistemic uncertainty types for tolerance analysis. There exist some approaches for the heterogeneous uncertainty propagation.

While information regarding variability is best conveyed using probability distributions, information regarding imprecision is more faithfully conveyed using families of probability distributions encoded either by probability-boxes (upper & lower cumulative distribution functions) or possibility distributions (also called fuzzy intervals) or yet by random intervals using belief functions of Shafer. Different theories have been used to handle epistemic uncertainty. The theories include probability theory and non-probability theories such as evidence theory, possibility theory, and fuzzy set theory [11].

A classical problem of heterogeneous uncertainty propagation can be represented by any scalar process variable or model outcome Y in terms of joint epistemic and aleatory uncertainties as follows:

$$Y = h(U,V)$$

where U = {all epistemic uncertainties (uncertain parameters)},

V = {aleatory uncertainties (stochastic variables)},

h is the computational model considered as a deterministic function of both uncertainties mentioned above.

Compared to the classical problem of heterogeneous uncertainty propagation, the statistical tolerance analysis problem does not consider the aleatory uncertainty and the epistemic uncertainty at the same level. To do so, we need to transform the probability expression into the probability of the worst cases due to the epistemic uncertainty:

$$P_A = P(AC) = P(\min_{G \in \{G \in \boldsymbol{R}^m : C_c(X,G) = 0 \cap C_{i^*}(X,G) = 0\}} (C_i(X,G)) \leqslant 0) \quad (3)$$

$$P_{FR} = P(FC) = P\left(\max_{G \in \{G \in \mathbf{R}^m : Cc(X,G) = 0 \cap Ci(X,G) \leq 0 \cap Ci^*(X,G) = 0\}} (C_f(X,G)) \leq 0\right) \quad (4)$$

In this work, the two types of uncertainty are analyzed. Aleatory uncertainty is modeled by probability distributions while epistemic uncertainty is modeled by intervals. Probabilistic analysis and interval analysis are integrated to capture the effect of the two types of uncertainty. The Monte Carlo Simulation is employed for probabilistic analysis while nonlinear optimization is used for interval analysis. The above process is called probabilistic analysis because only random variables are involved. As shown in equations (3) and (4), we need to find the maximum and minimum values. The process of finding the maximum and minimum is called interval analysis. Solving equations (3) and (4) directly requires a double-loop procedure where probabilistic analysis and interval analysis are nested.

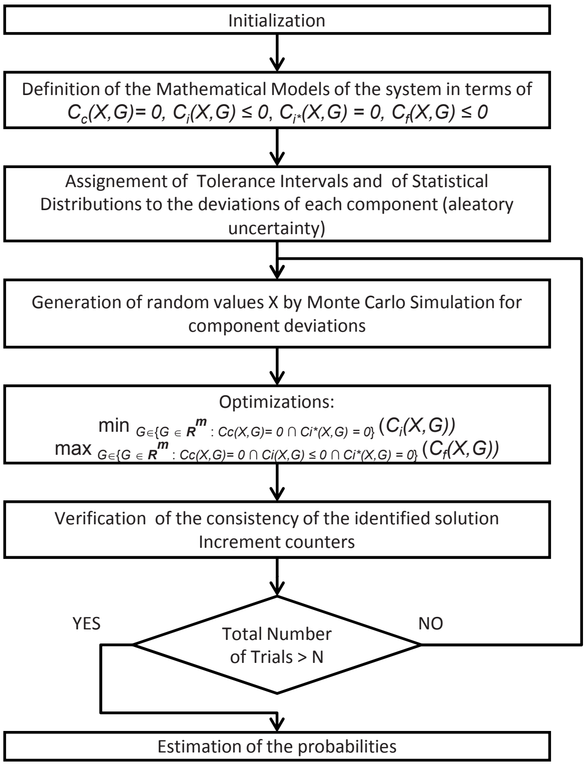

Fig. 2. General scheme of Tolerance analysis with Monte Carlo simulation.

An algorithm is proposed based on statistical sampling power of Monte Carlo simulation and on optimization to find the worst gap configuration. A general flow chart describing the module for tolerance analysis is shown in figure 2. The main principle behind the algorithm is to simulate and evaluate the influence of the manufacturing deviations on the nominal dimensions of an assembly. In order to achieve this, Monte Carlo simulation is used to simulate the deviations and the optimization is used to identify the worst gap configuration. This process is repeated recursively for a large sample of deviations to estimate assembly probability in order to perform the tolerance analysis of any given mechanism consisting of sub components.

This algorithm and its application are detailed in [7]. The example is the simplified version of a forging tool with omission of some components. Figure 3 illustrates the different views of the case study mechanism. The two main parts are assembled by three guide shafts. The contact between the shafts and the part 2 is fixed, and the contact between the shafts and the part 1 is floating. The functional characteristic is coaxiality between the center holes of the two parts. The algorithm was tested with 10,000 simulations for different nominal values and standard deviations:

Table 1. Simulation results.

Nominal dimensions (mm)	Standard deviation (mm)	Probability of AC (%)	Probability of AC & FC (%)
d1=20; d4=19,5	0,03	99,81	95,98
d1=20; d4=19,8	0,03	59,87	59,87
d1=20; d4=19,8	0,01	99,91	99,91

These formulation and approach have some similarities with the evidence theory [12]: belief and plausibility measures. They can be considered as the lower and upper bounds of a probability measure. In the case of unified uncertainty propagation, the outcomes of the uncertainty analysis are cumulative belief and plausibility functions (CBF and CPF) [13]. Traditional probabilistic analysis methods can be used for the unified uncertainty analysis.

5. Conclusion.

In the case of tolerancing, a balance must be made between a theoretically rigorous classification and a classification that can actually be implemented in a real-world setting. Based on this classification, a formulation of the tolerance analysis problem is proposed: the component deviations are aleatory and so irreducible (due to manufacturing imprecision, aleatory uncertainty exists in the geometrical component dimensions); and the gaps between components are epistemic uncertainty (due to the complexity of system behavior with gaps, epistemic uncertainty exists in the behavior model). The ultimate goal of this formulation is to develop methods for propagating and mitigating the effect of uncertainty that can be applied to any complex multidisciplinary engineering system for the tolerance analysis.

In fact, this new formulation has some similarities with others heterogeneous uncertainty propagation like the Probability Risk Analysis, the evidence theory, … It should be an area for some intense research to improve the uncertainty propagation techniques for the tolerance analysis.

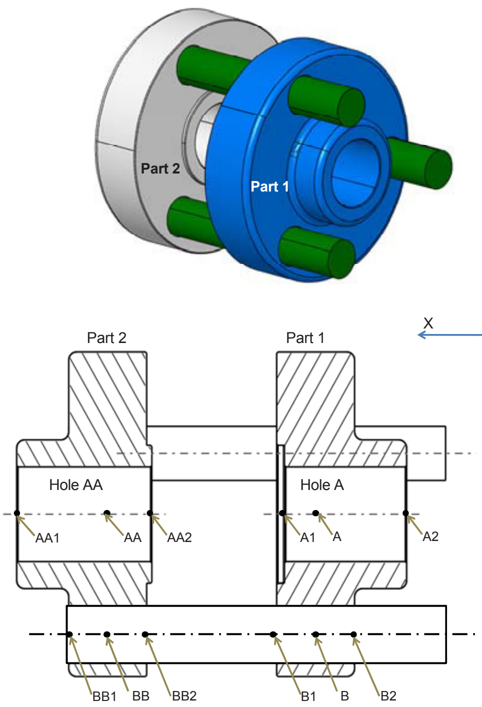

Fig. 3. Example.

Acknowledgements

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References

[1] OBERKAMPF, W. L., DELAND, S. M., RUTHERFORD, B. M., DIEGERT, K. V., AND ALVIN, K. F., "*Error and Uncertainty in Modeling and Simulation*," Reliability Engineering and Safety Systems, 75(3), pp. 333-357, 2002.

[2] NIKOLAIDIS, E., GHIOCEL, D. M., AND SINGHAL, S., *“Engineering Design Reliability Handbook”*, CRC Press, Boca Raton, Florida, 2005.

[3] HANS-GEORG BEYER, BERNHARD SENDHOFF, “*Robust optimization – A comprehensive survey*”, Computer Methods in Applied Mechanics and Engineering, Volume 196, Issues 33-34, 1 July 2007, Pages 3190-3218

[4] TANNERT C., ELVERS H.D., JANDRIG B., *“The ethics of uncertainty”,* EMBO REPORTS, VOL 8, NO 10, 2007

[5] THUNNISSEN D.P., “*Uncertainty classification for the design and development of complex systems*”, Proceedings of the 3 rd Annual Predictive Methods Conference, Veros Software, 2003.

[6] VESELY W.E., RASMUSON, D.M. *“Uncertainties in Nuclear probabilistic risk analyses.”* RISK ANALYSIS, VOL. 4. NO. 4, PP. 313-322, 1984.

[7] QURESHI J., DANTAN J., SABRI V., BEAUCAIRE P., GAYTON N., “*A statistical tolerance analysis approach for over-constrained mechanism based on optimization and Monte Carlo simulation”*, Computer Aided Design, Ed. Elsevier, Volume 44, Issue 2, February 2012, Pages 132-142.

[8] DANTAN J.Y., QURESHI J.; *“Worse Case and Statistical Tolerance Analysis based on Quantified Constraint Satisfaction Problems and Monte Carlo Simulation”*, Computer Aided Design, Ed. Elsevier, Vol. 41, N° 1, pp 1-12, 2009.

[9] QURESHI J., DANTAN J.-Y., BRUYERE J., BIGOT R., *“Tolerance Analysis based on Quantified Constraint Satisfaction Problems”*, 11th CIRP International Seminar on Computer Aided Tolerancing, 2009

[10] BALLU A., PLANTEC J.-Y., MATHIEU L., *“Geometrical reliability of overconstrained mechanisms with gaps”*. CIRP Annals – Manufacturing Technology 57:159-162, 2009.

[11] GUO J., DU X. . “*Sensitivity Analysis with Mixture of Epistemic and Aleatory Uncertainties*”, AIAA Journal, vol. 45, no. 9, pp. 2337-2349, 2007.

[12] SHAFER G., “*A Mathematical Theory of Evidence*”, Princeton, NJ, 1976.

[13] DU X., “*Uncertainty Analysis with Probability and Evidence Theories*” Proceedings of ASME 2006 International Design Technical Conferences & Computers and Information in Engineering Conference, Sep 10-13, Philadelphia, Pennsylvania, 2006.