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Two-dimensional numerical simulations of nonlinear acoustic streaming in standing waves

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Abstract

Numerical simulations of compressible Navier-Stokes equations in closed two-dimensional channels are performed. A plane standing wave is excited inside the channel and the associated acoustic streaming is investigated for high intensity waves, in the nonlinear streaming regime. Significant distortion of streaming cells is observed, with the centers of streaming cells pushed towards the end-walls. The mean temperature evolution associated to the streaming motion is also investigated.

Keywords: acoustic streaming, standing wave, numerical simulation, nonlinear streaming regime

1. Introduction

2 Acoustic streaming is generated inside a two-dimensional channel as a
3 consequence of the interaction between a plane standing wave and the solid
4 boundaries. It consists of a mean second order flow produced mainly by shear

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5 forces within the viscous boundary layer along the solid walls. This motion
6 was initially studied by Rayleigh [1] in the case of wide channels, in which the
7 boundary layer thickness is negligible in comparison with the channel width.
8 This streaming flow is characterized by four steady counter-rotating vortices
9 outside the boundary layer, nowadays referred to as Rayleigh streaming. The
10 vortices develop along the half wavelength of the standing wave. Along the
11 central axis of the channel, the streaming motion is oriented from acoustic
12 velocity nodes to antinodes. Inside the boundary layer four additional vor-
13 tices are created simultaneously, with the streaming motion oriented from
14 acoustic velocity antinodes to nodes along the inner walls of the tube [2, 3].

15 In the case of wide channels, Menguy and Gilbert [4] showed that stream-
16 ing itself can be linear (case of slow streaming) or nonlinear (case of fast
17 streaming), and both regimes are characterized by a reference nonlinear
18 Reynolds number $Re_{NL} = (M \times y_0/\delta_\nu)^2$ reflecting the influence of inertial
19 effects on the streaming flow (M is the acoustic Mach number, $M = U_{max}/c_0$,
20 with U_{max} the maximum acoustic velocity inside the channel and c_0 the ini-
21 tial speed of sound, y_0 is the half width of the channel and δ_ν the viscous
22 boundary layer thickness). Most analytical streaming models have been es-
23 tablished in the case of slow streaming, characterized by $Re_{NL} \ll 1$. They
24 are based on successive approximations of the nonlinear hydrodynamic equa-
25 tions and have been derived for arbitrary values of the ratio y_0/δ_ν , taking
26 into account the variations of heat conduction and viscosity with tempera-
27 ture [5], and the existence of a longitudinal temperature gradient [6]. In the
28 case $Re_{NL} = O(1)$, Menguy and Gilbert [4] derived an asymptotic model for
29 streaming flow inside wide cylindrical resonators, with no mean temperature

30 gradient, and showed a distortion of streaming patterns due to inertia ef-
31 fects. However, this model does not cover the strongly nonlinear streaming
32 regime ($Re_{NL} \gg 1$), and does not explain the nonlinear effects on acoustic
33 streaming recently observed in several experimental works [7, 8, 9], where
34 the temperature gradient along the resonator wall has a significant influence.

35 Numerical simulations in the linear regime, yielding results for non ide-
36 alized geometries, were performed in the specific cases of thermoacoustic
37 refrigerators [10] or in annular resonators [11] and solved the dynamics of
38 the flow without taking heat transfer into account.

39 Simulations in the nonlinear regime were first performed by Yano [12],
40 who studied the acoustic streaming associated with resonant oscillations with
41 periodic shock waves in tubes with aspect ratio (width over length) very
42 large (0.1). He solved the full 2D Navier-Stokes equations with an upwind
43 finite-difference TVD scheme and showed the existence of irregular vortex
44 structures and even turbulent streaming for high streaming Reynolds num-
45 bers (based on a characteristic streaming velocity, the tube length, and the
46 kinematic viscosity, $R_s = U_s L / \nu$). This is a different configuration than our
47 configuration, since it considers low frequency acoustic waves in wide tubes
48 with respect to their length and focuses on turbulent streaming.

49 Simulations of acoustic streaming in the linear and nonlinear regime, tak-
50 ing heat transfer into account, in a two-dimensional rectangular enclosure,
51 were performed by Aktas and Farouk [13]. In their study, the wave is created
52 by vibrating the left wall of the enclosure and the full compressible Navier-
53 Stokes equations are solved, with an explicit time-marching algorithm (a
54 fourth order flux-corrected transport algorithm) to track the acoustic waves.

55 Their numerical results are in agreement with theoretical results in the lin-
56 ear regime and show irregular streaming motion in the nonlinear regime,
57 but they show the existence of irregular streaming at small values of Re_{NL} ,
58 in contradiction with experiments cited above. Moreover, these simulations
59 do not analyze the deformation of the streaming cells until they split onto
60 several cells.

61 We propose in this work to conduct numerical 2D compressible simula-
62 tions for studying the origin of the distortion of streaming cells (of Rayleigh
63 type) that were experimentally observed. Calculations are performed for
64 channels with aspect ratios ranging from 0.01 to 0.07, and the coupling be-
65 tween streaming effects and thermal effects in the channel (existence of a
66 mean temperature gradient) is also investigated.

67 **2. Problem description and numerical model**

68 We consider a rectangular channel of length L and half width y_0 , initially
69 filled with the working gas. In order to initiate an acoustic standing wave in
70 the channel, it is shaken in the longitudinal direction (x), so that an harmonic
71 velocity law is imposed, $\mathbf{V}(t) = (V(t), 0)^T$, with $V(t) = x_p \omega \cos(\omega t)$, ω being
72 the angular frequency and x_p the amplitude of the channel displacement.
73 The channel being undeformable, the flow can be modeled by the compress-
74 ible Navier-Stokes equations expressed in the moving frame attached to the

75 channel, so that a forcing source term is added. The model reads:

$$\left\{ \begin{array}{l} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \\ \frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) + \nabla p = \nabla \cdot (\bar{\bar{\tau}}) - \rho \frac{d\mathbf{V}}{dt} \\ \frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho E \mathbf{v} + p \mathbf{v}) = \nabla \cdot (k \nabla T) + \nabla \cdot (\bar{\bar{\tau}} \mathbf{v}) - \rho \mathbf{v} \cdot \frac{d\mathbf{V}}{dt} \end{array} \right. \quad (1)$$

76 where $\mathbf{v} = (u, v)^T$ is the flow velocity, $E = e + \frac{1}{2} \mathbf{v} \cdot \mathbf{v}$ is the total energy,
77 with $e = \frac{p}{(\gamma-1)\rho}$ the internal energy, γ the specific heat ratio, $\bar{\bar{\tau}} = -\frac{2}{3}\mu(\nabla \cdot$
78 $\mathbf{v})\bar{\bar{I}} + 2\mu\bar{\bar{D}}$ the viscous stress tensor of a Newtonian fluid, $\bar{\bar{D}}$ the strain tensor,
79 μ the dynamic viscosity, k the thermal conductivity. The thermo-physical
80 properties μ and k are supposed to be constant. The gas is considered as
81 a perfect gas obeying the state law $p = r\rho T$, where T is the temperature
82 and r is the perfect gas constant corresponding to the working gas. The
83 physical boundary conditions employed in the moving frame are: no slip and
84 isothermal walls.

85 The model is numerically solved by using high order finite difference
86 schemes, developed in Daru and Tenaud [14]. An upwind scheme, third
87 order accurate in time and space, is used for convective terms, and a cen-
88 tered scheme, second order, is used for diffusion terms. More detail about
89 the scheme and computations showing its good qualities can be found in
90 Daru and Gloerfelt [15], Daru and Tenaud [16]. This scheme can be derived
91 up to an arbitrary order of accuracy for convective terms in the case of a
92 scalar equation. Here the third order scheme is selected, after having done
93 several comparisons using higher order schemes (up to the 11th order), that
94 have shown that third order gives sufficient accuracy for a reasonable CPU
95 cost. In cases where shock waves are present, the scheme can be equipped

96 with a flux limiter (MP), preserving monotonicity, intended for suppressing
 97 the parasitic numerical oscillations generated in the shock region, while pre-
 98 serving the accuracy of the scheme in smooth regions. However, the flows
 99 considered here are always low Mach number flows. Although traveling shock
 100 waves are a main feature of the flow for high acoustics levels, as noticed by
 101 several authors [17], they are of weak intensity and the numerical oscillations
 102 are very small and do not spoil the solution. Thus the MP limiter, which
 103 is expensive in terms of CPU cost, was not activated in these calculations.
 104 For solving the 2D Navier-Stokes equations, the scheme is implemented using
 105 Strang splitting. This reduces the formal accuracy of the scheme to second
 106 order. However, numerical experiments have shown that a very low level of
 107 error is still achieved.

108 Let us describe our numerical procedure. The system (1) can be written
 109 in vector form :

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial x}(f - f^v) + \frac{\partial}{\partial y}(g - g^v) = h \quad (2)$$

110 where w is the vector of conservative variables $(\rho, \rho u, \rho v, \rho E)^T$, f and g are
 111 the inviscid fluxes $f = (\rho u, \rho u^2 + p, \rho uv, \rho Eu + pu)^T$ and $g = (\rho v, \rho uv, \rho v^2 +$
 112 $p, \rho Ev + pv)^T$, f^v and g^v being the viscous fluxes $f^v = (0, \tau_{xx}, \tau_{xy}, k \frac{\partial T}{\partial x} +$
 113 $u\tau_{xx} + v\tau_{xy})^T$, $g^v = (0, \tau_{xy}, \tau_{yy}, k \frac{\partial T}{\partial y} + u\tau_{xy} + v\tau_{yy})^T$. The source term reads
 114 $h = (0, -\rho \frac{dV}{dt}, 0, -\rho u \frac{dV}{dt})^T$. Denoting $w_{i,j}^n$ the numerical solution at time $t =$
 115 $n\delta t$ and grid point $(x, y) = (i\delta x, j\delta y)$, we use the following Strang splitting
 116 procedure to obtain second order of accuracy every two time steps :

$$w_{i,j}^{n+2} = L_{\delta x} L_{\delta y} L_{\delta y} L_{\delta x} w_{i,j}^n \quad (3)$$

117 where $L_{\delta x}$ (resp. $L_{\delta y}$) is a discrete approximation of $L_x(w) = w + \delta t(-f_x +$

118 $f_x^v + h)$ (resp. $L_y(w) = w + \delta t(-g_y + g_y^v)$). The 1D operators being similar
 119 in the two directions, we only describe the x operator. The scheme is imple-
 120 mented as a correction to the second order MacCormack scheme. It consists
 121 of three steps, as follows :

$$\begin{aligned}
 w_{i,j}^* &= w_{i,j}^n - \frac{\delta t}{\delta x} (f_{i+1,j} - f_{i,j} - f_{i+1/2,j}^v + f_{i-1/2,j}^v)^n + \delta t h_{i,j}^n \\
 w_{i,j}^{**} &= w_{i,j}^* - \frac{\delta t}{\delta x} (f_{i,j} - f_{i-1,j} - f_{i+1/2,j}^v + f_{i-1/2,j}^v)^* \\
 w_{i,j}^{n+1} &= \frac{1}{2}(w_{i,j}^n + w_{i,j}^{**}) + C_{i+1/2,j}^x - C_{i-1/2,j}^x
 \end{aligned} \tag{4}$$

122 The viscous fluxes are discretized at each interface using centered second or-
 123 der finite differences formulae. The corrective term $C_{i+1/2,j}^x - C_{i-1/2,j}^x$ provides
 124 the third order accuracy and the upwinding for the inviscid terms. Let us de-
 125 fine $\psi_{i+1/2,j} = \frac{1}{6} \sum_{l=1}^4 \left\{ |\nu_{i+1/2,j}^l| (1 - \nu_{i+1/2,j}^l) (1 + \nu_{i+1/2,j}^l) \delta \alpha_{i+1/2,j}^l \cdot d_{i+1/2,j}^l \right\}$,
 126 where $\nu^l = \frac{\delta t}{\delta x} \lambda^l$, λ^l and d^l are the eigenvalues and eigenvectors of the Roe-
 127 averaged jacobian matrix $A = \frac{df}{dw}$ [18], and $\delta \alpha^l$ is the contribution of the
 128 l -wave to the variation $(w_{i+1/2,j}^n - w_{i-1/2,j}^n)$. Using the function ψ , the cor-
 129 rective term reads :

$$C_{i+1/2,j}^x = \begin{cases} -\psi_{i+1/2,j}^n + \psi_{i-1/2,j}^n & \text{if } \nu_{i+1/2,j} \geq 0 \\ \psi_{i+3/2,j}^n - \psi_{i+1/2,j}^n & \text{if } \nu_{i+1/2,j} < 0 \end{cases} \tag{5}$$

130 This completes the description of the numerical method.

131 We are interested in the acoustic streaming generated by the interaction of
 132 the imposed plane standing wave and the channel wall. Resonant conditions
 133 are imposed, for which $L = \lambda/2$, $\lambda = c_0/f$ being the wave length, c_0 the speed
 134 of sound for initial state and f the vibration frequency of the channel. It is
 135 known [5] that boundary layers develop along the walls, with thickness $\delta_\nu =$
 136 $\sqrt{2\nu/\omega}$, ν being the kinematic viscosity $\nu = \mu/\rho_0$, and ρ_0 the density at initial
 137 state. Depending on the value of the ratio y_0/δ_ν , several patterns of streaming

138 can appear: Rayleigh-type streaming in the central region, and boundary
 139 layer type streaming near the longitudinal walls. The boundary layer is of
 140 small thickness and must be correctly resolved by the discretization mesh.
 141 After several trials, we have determined that a value of 5 points per boundary
 142 layer thickness is sufficient for reasonable accuracy of the simulations. The
 143 results obtained using 10 points per boundary layer thickness show very small
 144 differences with the former, the maximum value of the differences being less
 145 than 3%. All results presented below are thus obtained using a cartesian
 146 mesh of rectangular cells of constant size δx and δy , composed of 500 points
 147 in the axial direction x , and of $5 \times y_0/\delta_\nu$ points in the y direction normal to
 148 the axis. In the considered geometry, this leads to cells such that $\delta y \ll \delta x$.
 149 The flow being symmetrical with respect to the x axis (at least in the range
 150 of parameters treated), only the upper half of the channel was considered.
 151 Also, the scheme being fully explicit, the time step δt is fixed such as to
 152 satisfy the stability condition of the scheme which can be written as:

$$\delta t \leq \frac{1}{2} \min(\delta y^2/\nu, \delta y^2/(k/\rho_0 c_0), \delta y/c_0) \quad (6)$$

153 As shown in Equation (6), the first two limiting values $\delta y^2/\nu$ and $\delta y^2/(k/\rho_0 c_0)$
 154 are related to the viscous and thermal conduction terms, and the third one
 155 $\delta y/c_0$ is related to the acoustic propagation. In all cases considered here,
 156 the time step limitation is acoustic, *ie* $\delta t \leq \frac{1}{2}\delta y/c_0$. Taking $\delta t = \frac{1}{2}\delta y/c_0$ and
 157 $\delta y = \delta_\nu/5$, this results in a number of time steps N_T per period of oscillation
 158 proportional to \sqrt{L} , $N_T = 1/(f\delta t) = 10\sqrt{\frac{2\pi c_0}{\nu}}\sqrt{L}$. Since transients of sev-
 159 eral hundreds of periods may be needed in order to reach stabilized steady
 160 streaming flow, simulations are very costly, and one must rely on numerical
 161 schemes that are sufficiently accurate in both space and time.

162 Finally, the mean flow is obtained from calculating a simple mean value
 163 for each physical quantity (velocity, pressure, temperature) over an acoustic
 164 period. The mean velocity obtained is the so-called Eulerian streaming ve-
 165 locity. The Lagrangian streaming velocity, associated to the streaming mass
 166 transport, could also be computed. The difference between them is signifi-
 167 cant only in the boundary layer, and in the case of wide channels the two
 168 velocities are almost the same. In order to observe the mechanism of cell
 169 distortion, either one of these velocities can be monitored.

170 **3. Numerical results**

171 We consider a channel initially filled with air at standard thermody-
 172 namic conditions, $p_0 = 101325\text{Pa}$, $\rho_0 = 1.2\text{kgm}^{-3}$, $T_0 = 294.15\text{K}$. The
 173 thermo-physical properties of air are $\mu = 1.795 \cdot 10^{-5}\text{kgm}^{-1}\text{s}^{-1}$ and $k =$
 174 $0.025\text{Wm}^{-1}\text{K}^{-1}$. Also for air, $\gamma = 1.4$ and $r = 287.06\text{Jkg}^{-1}\text{K}^{-1}$. The
 175 Prandtl number Pr is equal to 0.726. This results in an initial speed of
 176 sound $c_0 = 343.82\text{ms}^{-1}$.

177 For a 1m long channel, the limiting time step would correspond to $N_T \approx$
 178 $25000\sqrt{L}$, that is 25000 iterations per period. Since transients of several
 179 hundreds of periods may be needed in order to reach stabilized streaming
 180 flow, several millions of iterations are necessary for each simulation. Con-
 181 sidering these numerical constraints, a shorter channel is considered, with
 182 $L = 8.59\text{mm}$. This corresponds to a high-frequency wave, with $f = 20000\text{Hz}$.
 183 The resulting boundary layer thickness is $\delta_\nu = 1.54 \cdot 10^{-5}\text{m}$. The time step
 184 $\delta t = 8 \cdot 10^{-9}\text{s}$ is chosen in order to satisfy the numerical stability condition,
 185 corresponding to 6250 time iterations per period for a mesh involving 5 grid

$x_p(\mu m)$	5	10	50	80	100
$U_{max}(m/s)$	7.0	26.45	61.11	70.94	89.96
y_0/δ_ν	10	40	20	10	20
y_0/L	0.0180	0.0718	0.0359	0.018	0.0359
Re_{NL}	0.041	9.469	12.636	4.257	27.384

Table 1: Values of the parameters of the simulations.

186 points across the boundary layer thickness. The acoustic velocity produced
187 in the channel depends on the amplitude of the channel displacement and on
188 the ratio y_0/δ_ν . It varies approximately linearly with the amplitude of the
189 channel displacement, for a given ratio y_0/δ_ν . Table 1 summarizes the dif-
190 ferent parameter values corresponding to the simulations that are presented
191 thereafter.

192 As mentioned earlier, the parameter identified as relevant in describing
193 the regularity of streaming flow is the nonlinear Reynolds number Re_{NL}
194 introduced by Menguy and Gilbert [4]. In this paper we used a slightly
195 different definition for Re_{NL} , because the definition of the viscous boundary
196 layer thickness is different. Our Reynolds number corresponds to half of that
197 of Menguy and Gilbert [4].

198 We first present results concerning the main acoustic field in the channel,
199 for a small value of Re_{NL} corresponding to slow streaming. In Figure 1(a)
200 is represented the velocity signal at the center of the channel, as a function
201 of the number of periods elapsed. At this location, the acoustic velocity
202 amplitude is maximum since it corresponds to the antinode. For this value
203 of Re_{NL} , the problem is nearly linear and the final signal is purely sinu-

204 soidal, in agreement with the linear theory. The amplification of the initial
 205 perturbation until saturation can be observed. The periodic regime is estab-
 206 lished after about 20 periods. Figure 1(b) shows the time evolution of the
 207 mean horizontal velocity (over an acoustic period) and of the mean temper-
 208 ature difference $\Delta T = T - T_0$ (also over an acoustic period) on the axis,
 209 at $x = \lambda/8$. At this location the streaming velocity is maximum. It can be
 210 noticed that the steady streaming field is established also after about 20 peri-
 211 ods which is of the same order of magnitude as the theoretical characteristic
 212 streaming time scale $\tau_c = (\frac{2y_0}{\pi})^2 \frac{1}{\nu}$, (see Amari, Gusev and Joly [19]) which
 213 in this case gives n_c periods for reaching steady-state, with $n_c = 13$. In Fig-
 214 ure 2(a) is shown the variation of the axial dimensionless streaming velocity
 215 at $x = \lambda/8$ along the channel's width, compared with results computed us-
 216 ing the analytical expressions of Hamilton, Ilinskii and Zabolotskaya [20]. In
 217 this figure, the reference velocity is the Rayleigh streaming reference velocity
 218 [2, 5], $u_{Rayleigh} = \frac{3}{16} U_{max}^2 / c_0$. The slight discrepancy between the numerical
 219 and the analytical profiles is probably due to the presence of the vertical
 220 end walls, which is not accounted for in the model of Hamilton, Ilinskii and
 221 Zabolotskaya [20]. In Figure 2(b) is shown the stabilized mean pressure $p - p_0$
 222 (over an acoustic period), scaled by $(\gamma/4)p_0 M^2$, along the channel's axis. It
 223 is the second order average pressure resulting from the streaming flow, which
 224 is clearly one-dimensional and has a cosine variation with respect to x , as
 225 expected in the linear regime of streaming. In the present case, there is an
 226 offset pressure p_{off} , corresponding to an increase of the mean pressure and
 227 temperature (uniform in space) inside the channel, due to the harmonic forc-
 228 ing source term. When subtracting off this offset pressure, the theoretical

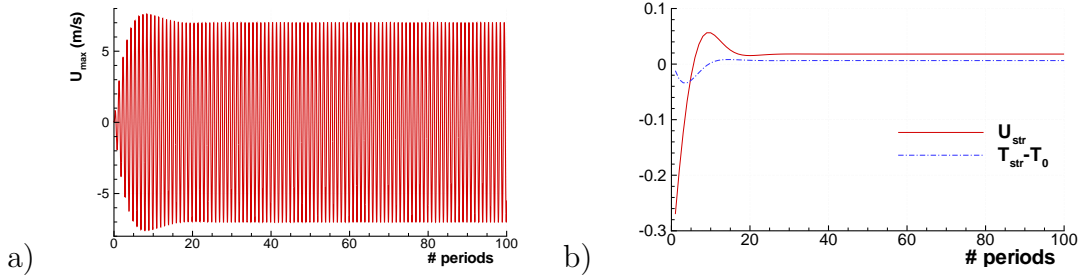


Figure 1: a) Acoustic velocity at the channel's center, as a function of time counted by the number of periods elapsed. b) Mean horizontal velocity and mean temperature variation, on the channel's axis at $x = \lambda/8$. Case $Re_{NL} = 0.041$ ($y_0/\delta_\nu = 10$, $M = 0.02$).

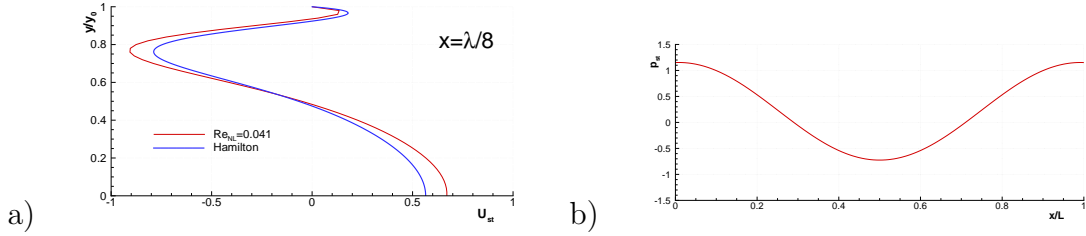


Figure 2: a) U_{st} as a function of y/y_0 at $x = \lambda/8$, numerical (present study) and analytical [20] results. b) Dimensionless mean fluctuating pressure, $p - p_0$ along the channel's axis. Case $Re_{NL} = 0.041$ ($y_0/\delta_\nu = 10$, $M = 0.02$).

229 result for the dimensionless hydrodynamic streaming pressure is obtained,
 230 $P_{2s} = \cos 4\pi \frac{x}{\lambda}$ (see Menguy and Gilbert [4]).

231 Simulations are then performed for several values of Re_{NL} correspond-
 232 ing to configurations ranging from slow streaming flow ($Re_{NL} = 0.041$) to
 233 fast streaming flow ($Re_{NL} = 27.384$), for several values of the cavity width
 234 ($y_0/\delta_\nu = 10, 20, 40$), and for increasing acoustic velocities, with Mach num-
 235 bers ranging from $M = 0.02$ to $M = 0.27$ so that shock waves can occur.
 236 This can be seen in Figure 3(right) showing the acoustic velocity signal at
 237 channel's center as a function of time counted by the number of periods

238 elapsed. In Figure 3(a)(right) the signal contains only one frequency, but for
 239 all other cases, there are shock waves and the acoustic velocity signal is dis-
 240 torted in a "U" shape, because of the presence of odd harmonics (3, 5, etc).
 241 Figure 3(left) shows the streamlines of the streaming velocity field over the
 242 whole length and only over the top half width of the channel. As expected, in
 243 the case of small Re_{NL} number values (Figure 3(a)), four symmetric stream-
 244 ing cells develop over the length and the half width of the channel: two cells
 245 in the boundary layer, and two cells in the core of the channel, identified in
 246 the literature as Rayleigh streaming. These results are in agreement with
 247 the predictions of analytical models of streaming flows [5, 6], and with ex-
 248 perimental measurements [7]. The only noticeable difference is the slight
 249 asymmetry of cells with respect to the vertical lines $x = \lambda/8$ and $x = 3\lambda/8$,
 250 due to the presence of vertical boundary layers. Indeed these boundary layers
 251 are accounted for in the present simulations but are neglected in the analyt-
 252 ical models, and are very far from the measurement area in the experiments.
 253 Several simulations have shown that this asymmetry is independent of Re_{NL}
 254 as long as the value of the latter remains small with respect to 1.

255 For $Re_{NL} > 1$, the steady streaming flow is established after the same
 256 characteristic time as in the linear case. The recirculation cells become very
 257 asymmetric as Re_{NL} increases, and streaming flow becomes irregular (Fig-
 258 ure 3(b,c,d,e)(left)). This was also observed experimentally (with PIV mea-
 259 surements) by Nabavi, Siddiqui and Dargahi [8] in a rectangular enclosure.
 260 The centers of all streaming cells (boundary layer cells as well as central
 261 cells) are displaced towards the ends of the resonant channel, close to the
 262 boundary layers next to the vertical walls. PIV measurements by Nabavi,

263 Siddiqui and Dargahi [8] show the same distortion of streamlines between an
 264 acoustic velocity node and an antinode. Figure 4(a) shows the x variation
 265 along the channel's central axis $y = 0$, of the axial dimensionless streaming
 266 velocity component, using as reference velocity the Rayleigh streaming ref-
 267 erence velocity [2, 5], $u_{Rayleigh} = \frac{3}{16}U_{max}^2/c_0$. There is a clear modification of
 268 the velocity profiles as Re_{NL} increases: the sine function associated to slow
 269 streaming becomes steeper next to the channel's ends. The slope to the curve
 270 at the channel's center (acoustic velocity node) becomes smaller as Re_{NL} in-
 271 creases, then becomes close to zero (curve parallel to the longitudinal axis) for
 272 a critical value between 13 and 27, and then changes sign, which indicates the
 273 emergence of new streaming cells (Figure 4(a)). Another consequence of the
 274 distortion of streaming cells can be observed on the acoustic streaming axial
 275 velocity profiles along the width of the channel, shown in Figure 4(b,c,d).
 276 The parabolic behavior in the center of the channel at $x = \lambda/8$ disappears
 277 as Re_{NL} increases (see Figure 4(b)), as a consequence of displacement of the
 278 center of each streaming cell toward the velocity node. Figures 4(c,d) also
 279 confirm the direction of the displacement of the streaming cells' centers. This
 280 distortion of streaming cells was already observed in experiments in rectan-
 281 gular or cylindrical geometries in wide channels [7, 8, 9]. Nabavi, Siddiqui
 282 and Dargahi [8] described it as irregular streaming and detected a critical
 283 nonlinear Reynolds number $Re_{NL} = 25$ that separates regular and irregu-
 284 lar streaming, which is in agreement with our simulations. In the literature
 285 there is to our knowledge no other theoretical or numerical study confirming
 286 measurements in these streaming regimes. With the weakly nonlinear model
 287 of Menguy and Gilbert [4] the streaming can be calculated for a maximum

288 value of $Re_{NL} = 2$ (in our definition), while the numerical simulations of
289 Aktas and Farouk [13] show the existence of multiple streaming cells for a
290 low value of $Re_{NL} = 1.4$, which is in contradiction with our results and with
291 experiments. Moreover, these numerical simulations [13] do not analyse in
292 detail the transition from two exterior streaming cells to more streaming
293 cells.

294 According to Menguy and Gilbert [4], the fluid inertia causes distortion
295 of streaming cells for large values of Re_{NL} . This was also verified through our
296 simulations. For $Re_{NL} = O(1)$, the Mach number is still small (the wave is
297 almost a mono-frequency wave) and the mean temperature difference inside
298 the channel is smaller than 0.1K (the mean temperature gradient is negligi-
299 ble). The approximations of the model by Menguy and Gilbert [4] still apply
300 here, so we can say that the distortion is caused only by inertial effects. When
301 Re_{NL} increases, periodic shocks appear and the mean temperature gradient
302 becomes important in our simulations. In their experimental study, Thomp-
303 son, Atchley and Maccarone [9] show the existence of some distortion of the
304 streaming field that are not predicted by existing models of the literature in
305 the nonlinear regime. They do not relate this distortion to fluid inertia but
306 rather to the influence of the mean temperature field, and more specifically
307 of the axial temperature gradient induced through a thermoacoustic effect
308 along the horizontal walls of the resonating channel. In an experimental case
309 with no shock waves, Merkli and Thomann [17] showed that a mean tem-
310 perature gradient is established inside the tube so that heat is removed close
311 to the velocity antinodes, i.e. at the location of largest viscous dissipation,
312 and heat is produced close to velocity nodes, along the lateral walls. Similar

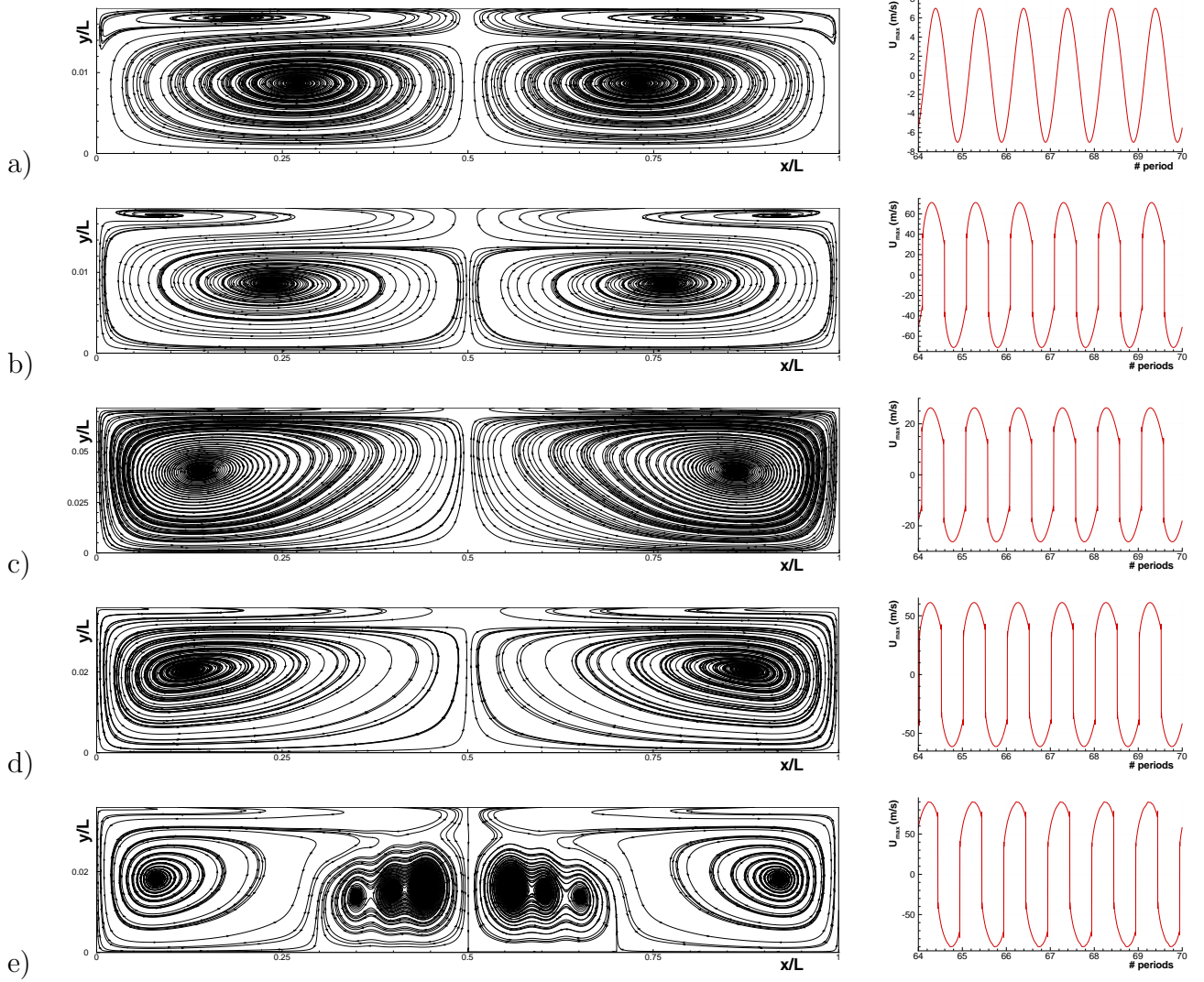


Figure 3: Streamlines of mean flow on the top half of the channel (left) and acoustic velocity signal at channel's center as a function of time counted by the number of periods elapsed (right) a) $Re_{NL} = 0.041$ ($y_0/\delta_\nu = 10$, $M = 0.02$). b) $Re_{NL} = 4.257$ ($y_0/\delta_\nu = 10$, $M = 0.206$). c) $Re_{NL} = 9.469$ ($y_0/\delta_\nu = 40$, $M = 0.077$). d) $Re_{NL} = 12.636$ ($y_0/\delta_\nu = 20$, $M = 0.178$). e) $Re_{NL} = 27.384$ ($y_0/\delta_\nu = 20$, $M = 0.262$). Lengths are normalized with L .

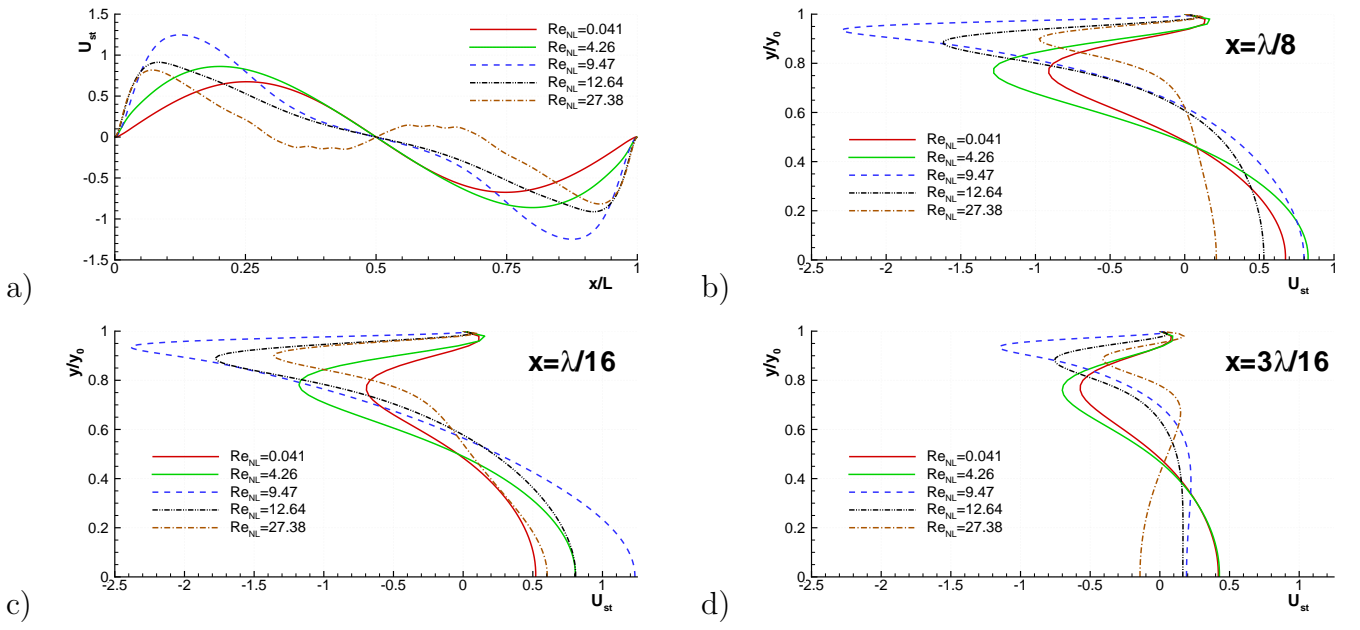


Figure 4: Horizontal mean velocity component U_{st} , normalized with $\frac{3}{16}U_{max}^2/c_0$ for the 5 cases of Figure 3. a) U_{st} along the channel's central axis. b),c) and d) U_{st} as a function of y/y_0 for several sections x .

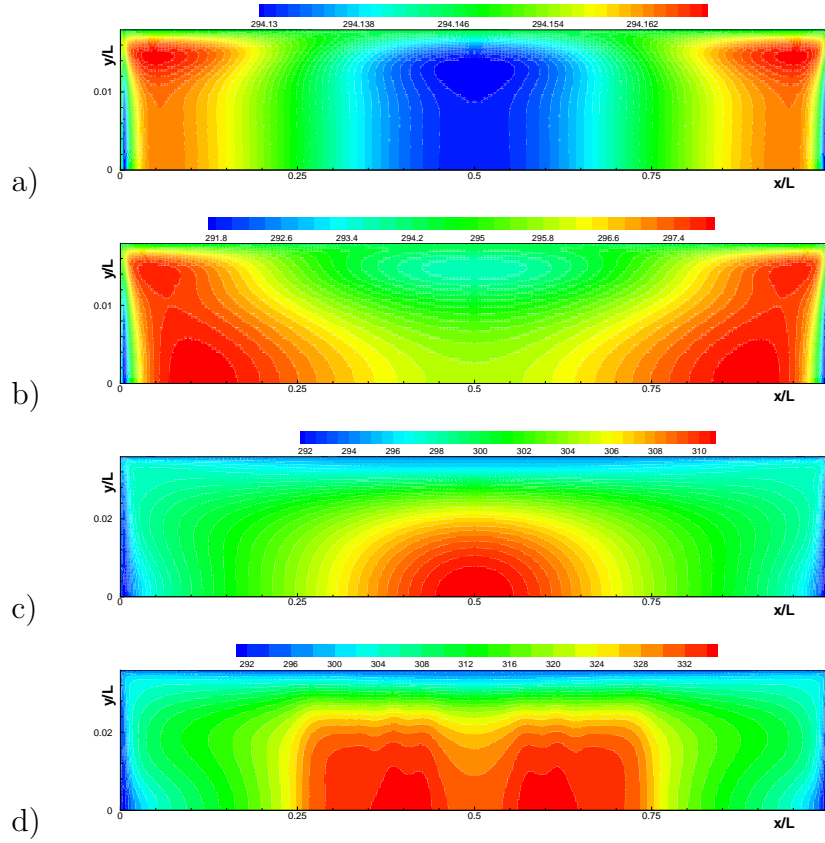


Figure 5: Mean temperature field on the top half of the channel, a) $Re_{NL} = 0.041$. b) $Re_{NL} = 4.257$. c) $Re_{NL} = 12.636$. d) $Re_{NL} = 27.383$. The difference between minimum and maximum values of temperature is (respectively) : a) $\Delta T = 0.039\text{K}$, b) $\Delta T = 6.39\text{K}$, c) $\Delta T = 19.4\text{K}$, d) $\Delta T = 44\text{K}$.

313 observations can be made in our simulations as seen in Figure 5(a) which
 314 shows the mean temperature field for small values of Re_{NL} . The thermoacoustic
 315 heat transport takes place at a distance of one thermal boundary
 316 layer thickness and then heat diffuses in the radial direction, yielding a tem-
 317 perature field almost one-dimensional in the central part of the tube in the
 318 steady-state. As Re_{NL} increases however, the mean temperature field clearly
 319 becomes two-dimensional, as a consequence of both convective heat trans-
 320 port by streaming flow and heat conduction in both directions (Figure 5(b)).
 321 Within the considered range of values of the nonlinear Reynolds number,
 322 there is a change of regime for the temperature field before $Re_{NL} = 13.26$,
 323 corresponding to the confinement of outer streaming cell towards the acous-
 324 tic velocity node. Consequently a zone of very small streaming velocities is
 325 generated in the middle of the cavity and that induces the accumulation of
 326 heat (Figure 5(c)). The mean temperature gradient changes the orientation
 327 and can cause the splitting of the outer cell into several cells when further
 328 increasing Re_{NL} (Figure 5(d)).

329 Note that the streaming flow stabilizes in several stages in regimes with
 330 high values of the nonlinear Reynolds number. In a first and rapid stage
 331 (a few tens of periods), regular streaming flow appears. Then this regular
 332 streaming is destabilized along with increasing heterogeneity of the mean
 333 temperature field. The steady mean flow stabilizes much later, with time
 334 scales related to convection and heat conduction.

335 **4. Conclusions**

336 The numerical simulations performed demonstrate the transition from
337 regular acoustic streaming flow towards irregular streaming, in agreement
338 with existing experimental data. These are the first simulations, to our
339 knowledge, in good alignment with experiments of nonlinear streaming regimes.
340 Results show a sizable influence of vertical boundary layers for the chosen
341 configuration. There is also intricate coupling between the mean tempera-
342 ture field and the streaming flow. This coupling effect will be the object of
343 future work. Also, extension of current results in configurations with larger
344 channels is currently in progress.

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348 **References**

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