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Hussein ZAHR, Eric SEMAIL, Franck SCUILLER - Five-phase version of 12slots/8poles three-phase Synchronous Machine for Marine-propulsion - In: IEEE Vehicle Power and Propulsion Conference, Portugal, 2014-10 - Five-phase version of 12slots/8poles three-phase Synchronous Machine for Marine-propulsion - 2014

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# Five-phase version of 12slots/8poles three-phase Synchronous Machine for Marine-propulsion

Hussein Zahr<sup>(1,2)</sup>, Eric Semail<sup>(1)</sup>

(1) L2EP, Electrical Engineering and Power  
Electronics Laboratory  
Arts et Metiers Paris Tech  
8, Boulevard Louis XIV Lille, France  
[Hussein.ZAHR@ensam.eu](mailto:Hussein.ZAHR@ensam.eu)  
[Eric.SEMAIL@ensam.eu](mailto:Eric.SEMAIL@ensam.eu)

Franck Scuiller

(2) Naval Academy Research Institute  
Brest, France  
[franck.scuiller@ecole-navale.fr](mailto:franck.scuiller@ecole-navale.fr)

**Abstract**— Multiphase machines are widely used in electric marine propulsion especially because of their fault-tolerance which allows to guarantee the propulsion even if a fault occurs in the electrical system. Besides, the 12 slots/8 poles three-phase machine (12/8/3 machine) with fractional slot concentrated windings is known for its low level of Permanent Magnet eddy currents losses, making it adequate for compact high speeds applications. Since this interesting property is due to a 0.5 value for the number of slots per pole and per phase ( $spp=0.5$ ), then the paper examines a five-phase 20 slots/8poles (20/8/5 machine) fault tolerant machine whose  $spp=0.5$ . Using an analytical model, the copper losses and an estimation of the magnet losses for the two machines are presented and a comparison is done between the two machines. The results show that the 20/8/5 has more Joules losses than 12/8/3 but lower magnet eddy current losses. Since the Joules losses can be easily evacuated more than the magnet losses, the 20/8/5 machine can be considered as the fault tolerant version of the 12/8/3 machine. Finally, the overall losses for the two machines are computed, the losses in 20/8/5 machine are less than in 12/8/3 machine. A finite element calculation is carried out in order to validate the analytical predictions.

**Keywords**—Concentrated windings, electrical propulsion, copper losses, eddy-current losses, Permanent magnet machines.

## I. INTRODUCTION

Nowadays, electrical machines are widely used in the automotive and marine propulsion. In these applications, the reliability of the propeller is required to ensure the functioning in fault mode. In addition, given the limited volume dedicated to the propeller, the propulsion must have a high compactness. Multiphase machines have the ability to satisfy these requirements, while providing other benefits: low torque ripple, the ability to produce a significant torque using high order harmonic injection to enhance torque density [1-6] and reducing the current per phase by splitting the same power over a greater number of phases reducing the constraints on power electronics [7]. Regarding the machine design, two important criteria must be adapted to obtain the desired performance for multiphase machine: winding distribution and the geometry of permanent magnet layer. However, winding distribution is determined by the winding configuration and slot/pole combination. Due to its advantages and its simple structure which is easy to manufacture and recycle, concentrated

winding are chosen as winding configuration [8][9]. With reference to distributed winding [2][3][4][6], this configuration can enhance the third harmonic term in the MMF spectrum. Thus, this harmonic, if exploited, can increase significantly the torque density of the machine[5]. The main drawback of such configuration is the magnetomotive force (MMF) spectrum which contains harmonics moving asynchronously with the rotor, inducing eddy-current losses in the different rotor parts, in particular permanent magnets [8][10][11]. The comparison introduced in [11] indicates that the rotor losses depend on slots/poles combination. These rotor losses, which are difficult to evacuate, heat the magnets whose magnetic properties are highly depending with the temperature. Thus, demagnetization phenomena can occur degrading the machine performances. As consequence, the choice of the slot/pole combination is important to determine the MMF harmonic content which impacts the amount of the rotor losses at high speed [12][13]. The comparison between the different combinations slots/poles/phases in order to avoid bad choice of this combination is introduced in [12][14][15][16]. A general analytical model of magnet eddy-current losses is developed and the average value of magnet losses for each combination slots/poles/phase is computed [16]. The results indicate that the combination 0.5 slot/pole/phase does not produce any sub harmonic in the MMF spatial spectrum, so this combination appears as the combination with the lower level of magnet losses. It is a reason why such 12/8/3 machine are used in hybrid automotive applications developed by Toyota and Honda car-makers.

Besides, in low voltage (48V) applications with significant power (>10kW), the phase currents in a three-phase machine become too high for standard power components and connections. If a low voltage level is preferable (pleasure boat or automotive) because it is a security level of voltage, then the only solution to decrease the phase current is to increase the phase number. Furthermore multi-phase machines are all the more relevant as it improves the torque density and reduces the torque ripples. Finally, when a fault-tolerance capability is also appreciated or required, such as in electrical marine applications, then multiphase machines become necessary.

A five-phase machine with  $s_{pp}=0.5$  can be a solution for operating at low voltages and high frequencies with a low level of eddy-current losses in Permanent Magnets.

In this paper, a 20slots/8poles five-phase machine with  $s_{pp}=0.5$  is chosen despite its low (0.588) first harmonic winding factor. The reason is that, regarding the control side, star-connected multiphase machines present particular properties which give more design and control possibilities than the three-phase machines. Thus, it is common to consider a five-phase machine as a set of two-phase virtual machines electrically independent but magnetically coupled (multi-machine decomposition) [17][18]. The two (d,q) type machines, characterized by different harmonic families (associated respectively with first and third harmonics), can be controlled independently. As consequence, when considering a five-phase machine, the third harmonic current injection can boost the torque which increases the machine torque density [2]. It should be mentioned that the third harmonic current injection can be used to increase the torque of multiphase induction machine. For example, according to [5], with reference to a standard three-phase induction machine, it is shown that this strategy applied to a six-phase induction machine equipped with concentrated winding can increase the torque by 40%. In [3], when using this control strategy, the gain is about 10 % with a five-phase induction motors referring to a standard three-phase one (with sinus supply). In [6] that focuses on five-phase PM machines, a torque increase by 17 % is shown with reference to a three-phase PM machine. These torque enhancements are all obtained for multi-phase machines equipped with concentrated windings in order to have a significant third harmonic term in the magnetomotive force. Finally, for a PM five-phase machine, the torque enhancement will be all the higher as the back-emf has a significant third harmonic term. This particularity for the third harmonic winding factor allows, with an adapted PM rotor design, to use the third harmonic of current to increase the torque, particularly at low speed. This is the original advantage of this machine over the 12/8/3 machine which can be supplied only with the first harmonic current.

This paper deals with the comparison between this two kinds of machines, the last being intended to drive ship propeller, typically for pleasure boat with 48V battery packs with a fault tolerance property.

In section II, the system is described with a focus on the two rotor magnet configurations for the two kinds of machine.

In section III, copper losses are calculated and compared for a given torque and different strategies of supply.

In section IV, at first the magnet losses are evaluated and compared in the same conditions as in section III.

In section V, a global comparison is achieved concerning the losses showing equivalent global losses.

## II. DESIGN SPECIFICATION OF SHIP PROPELLER AND MACHINE CONFIGURATIONS

The chain of naval propulsion is shown in figure (1):

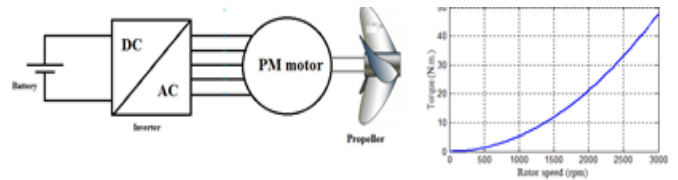


Fig. 1. Chain of electrical naval propulsion.

In steady state operation, the propeller follows a profile such that the load torque is proportional to the square of the speed[19]:

$$T_{shaft} = K\Omega^2 \quad (1)$$

Where  $T_{shaft}$  is the propeller torque and its rotation speed. This propeller has a base speed of 3000 rpm and the power at this speed is 9kW. As mentioned in the introduction, we have two possibilities for the electromagnetical design: the 12/8/3 or the 20/8/5. The two machines have the same geometrical parameters. The basic parameters of the propulsion machine are given in table (I):

TABLE I. BASIC PARAMETERS OF THE PROPELLER

Machine	12/8/3	20/8/5
Point de base	9kW @ 3000tr/min	
Effective length	280mm	
Outer diameter	140mm	
Stator yoke thickness	7.8mm	
Slot depth	6.4mm	
Air-gap length	1mm	
Magnet layer thickness	3mm	
Rotor thickness yoke	7.8mm	
Slot width to slot pitch ratio	0.5	
Slot opening to slot pitch ratio	0.5	
Filling factor	0.8	

As mentioned previously, fractional slot concentrated winding permanent magnet synchronous machines are good solutions for this application. Bearing in mind that the machine performance depends on the stator and rotor configuration, we try to adapt the magnet layer to the winding in order to improve the performance. Concerning the winding, the configurations with  $S_{pp}=0.5$  are favored. It should be noted that the winding distribution is chosen to maximize the winding factor of the first harmonic for 12/8/3 and for the first and the third harmonic for 20/8/5, where this choice allows to enhance the torque density, by injecting the third harmonic. Furthermore, this configuration allows the 20/8/5 machine to operate at low speed (3p pairs of pole) or at high speed (p pairs of pole) with an electronic pole changing effect. The second topic in this part is the choice of magnet layer geometry for the two machines under consideration (the 12/8/3 and the 20/8/5). The 12/8/3 machine has a zero winding factor for the third harmonic. Therefore, we decide to design the magnet layer to eliminate

the air gap flux density third harmonic, thus reducing the cogging torque. The pole arc to pole pitch ratio is then chosen to be 2/3. For the 20/8/5 machine, the same magnet volume as for the 12/8/3 machine must be used. Taking into account the winding factors of the first and the third harmonic, the magnet layer shape is arranged in order to have the same torque to current contribution when using the first harmonic current component or when using the third harmonic one. This objective implies the first harmonic (E1) and the third harmonic (E3) terms of the back-emf must be equal. In order to increase the third harmonic term of the air gap flux density, an interpolar gap  $x$  is introduced in each pole as in the figure (2):

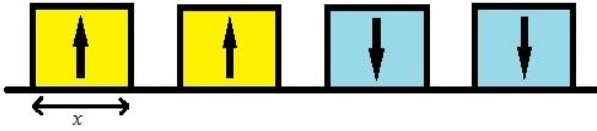


Fig. 2. Magnet layer of 20/8/5 machine .

Let  $K$  the ratio of magnet width  $x$  over the polar pitch. The better choice is  $K=1/3$  to obtain two virtual machines with the same torque production ability. In order to prove the validity of this solution the back-emf of 20/8/5 machine is calculated analytically. The results are validated with finite elements calculations. Figure (3) shows the comparison between the two methods: quite close estimations are obtained.

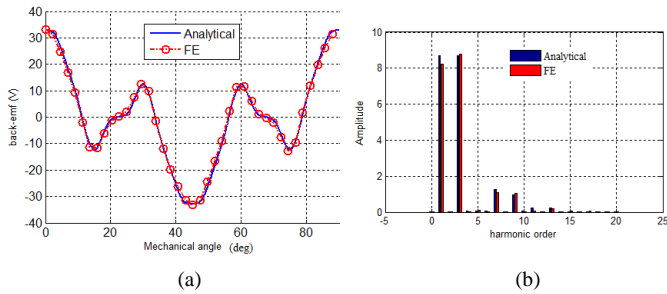


Fig. 3. (a) Back-emf for 20/8/5 machine . (b) Harmonic content of back-emf

The winding distributions over a pole pair for the two machines are drawn in figure(4):

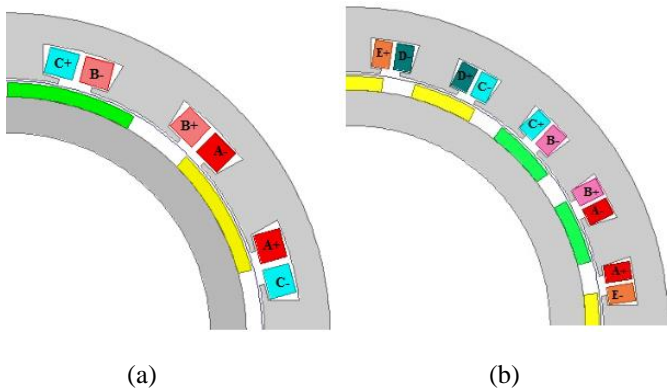


Fig. 4. (a) 12/8/3 machine . (b) 20/8/5 machine.

### III. COPPER LOSSES

In many cases, Copper losses are the most dominant losses in electrical machines at low electrical frequency. They are divided into DC losses and AC losses due to skin effect. This paper deals only with DC losses: the skin effect is neglected. In addition, the end windings effect is disregarded. Consequently, copper losses are given by (2):

$$P_j = mRI^2 \quad (2)$$

Where  $m$  is the number of phases,  $R$  is the resistance of coils and  $I$  is the RMS current. In this section, a comparison between the copper losses of 12/8/3 and 20/8/5 is carried out. The 12/8/3 machine is supplied with sinusoidal current. For the 20/8/5 machine, in order to obtain the base torque with the base current, we can control the machine currents in three ways: with only fundamental, with only third harmonic or with both fundamental and third harmonic. The following analysis is made with Maximum Torque per Ampere (MTPA) control assumption applied to the three strategies. The expression of the maximum torque is given by:

$$T_{em} = m \frac{\sum E_h I_h}{2\Omega} \quad (3)$$

Where  $h \in 2N+1$  and  $h < m$  ( $h=1$  if  $m=3$ ,  $h=1$  or  $h=3$  if  $m=5$ ). Thus, for 12/8/3 machine, the torque is produced by the first harmonic only, while the 20/8/5 can be supplied by the first harmonic, the third or both. The required currents corresponding to the base torque for the two machines are given in table (II), the calculation is done analytically and the results are validated numerically.

TABLE II. RMS CURRENTS INJECTED AT BASE SPEED AND TORQUE

Unit A	Machines					
	20/8/5					12/8/3
	First harmonic	Third harmonic	Both first and third harmonic			First harmonic
Supply			first	third	both	
Ana	207	208	104	103.5	146,7	193
Num	219	205	102.2	109.3	149,6	206.5

This difference between the currents values in the two cases can be explained by the difference between the analytical and FE estimations of the back-emf. The copper losses are the calculated in each case, according to the torque versus speed profile given by (1) and illustrated in figure (1). The results are shown in figure (5).

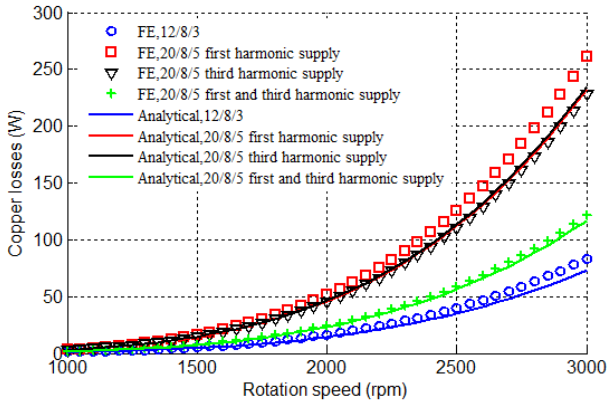


Fig. 5. Copper losses in the two machines 20/8/5 and 12/8/3.

According to figure(5), the 12/8/3 has the lower copper losses because of its higher fundamental winding factor. The supply of 20/8/5 machine with both the first and the third harmonic current harmonics reduce by half the copper losses. This is due to high winding factor of third harmonic (0.951) which compensates the additional losses. Therefore, the supply of 20/8/5 with these two harmonics is the better strategy. We can note that the slight difference between numerical and analytical calculations is due mainly to the difference between the analytical and numerical estimations of the back-emf, thus resulting to slight different analytical and numerical estimations of the required currents to obtain the wanted mean torque (as already observed in table (II)).

#### IV. MAGNET LOSSES

Different phenomena can induce eddy-current losses in the rotor: asynchronous MMF harmonics, permeance variation and PWM harmonics [12]. In this paper we consider the eddy-current losses due to the MMF harmonics associated with the windings and those due to permeance variation. The MMF losses, called armature reaction PM losses depend on three factors [10][12]:

- The amplitude of the magnetic flux density in the air-gap for the  $\nu$ -th MMF harmonic denoted  $B_\nu$
- The relative speed of the MMF harmonics with respect to the rotor
- The ratio  $\lambda_\nu/w$ : where  $w$  is the pole magnet width and  $\lambda_\nu = 2\pi R_r/\nu$ ,  $R_r$  being the rotor radius.

The second type of the losses, called open circuit PM losses, is the permeance variation losses, which is induced by the variation of permanent magnet field due to slot effect. This type of losses depends generally on the slot-opening width. In this section we will examine these two types of losses. An analytical estimation for each type is carried out; the results are validated with FE estimation.

#### A. Open circuit PM losses

The estimation of open circuit permanent magnet losses is mentioned in [20] where an analytical method is developed to estimate these losses. The origin of these losses is the variation of PM magnetic flux density due to stator slot effect. Consequently, this losses depend generally on slot opening. Applying this method to the two machines 20/8/5 and 12/8/3, we obtain the results illustrated in figure (6).

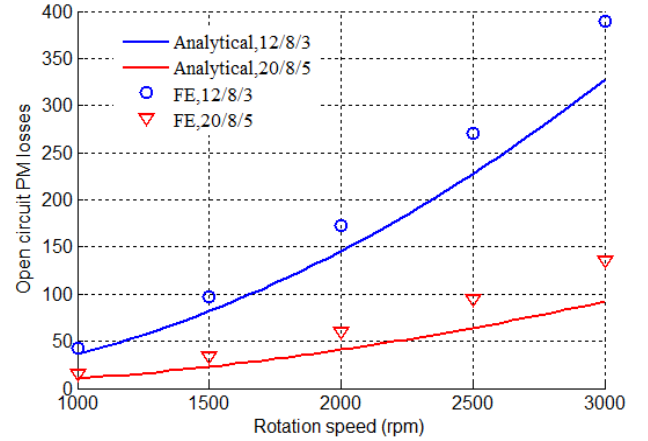


Fig. 6. Open circuit PM losses.

Figure (6) shows that analytical and numerical estimations are quite close. According to the two methods, the 12/8/3 has more open circuit PM losses than of 20/8/5. This is due to the bigger slot opening in 12/8/3 machine, since the 20/8/5 machine has more slots for the same stator diameter. In the next section, we will investigate the magnet losses due to armature reaction in order to calculate the total losses in magnet losses.

#### B. Armature reaction PM losses

The armature reaction PM losses is mainly due to the MMF space harmonics. These components move asynchronously with the rotor, inducing eddy currents in the magnet. The MMF harmonic content depends on the machine supply. We remind that the 20/8/5 can be supplied by either the first or the third current harmonic or both, whereas 12/8/3 can be supplied with only the first harmonic. Each strategy of supply generates a particular MMF spectrum. Table (III) presents the MMF harmonics for each supply strategy for the two machines (where  $c$  is an integer).

TABLE III. MMF SPECTRUM ACCORDING TO THE SUPPLY STRATEGY

Supply	Machines	
	20/8/5	12/8/3
First Harmonic	$p(5c\pm 1)$	$p(3c\pm 1)$
Third harmonic	$p(5c\pm 3)$	-----
First and third harmonic	$p(5c\pm 1)$ and $p(5c\pm 3)$	-----

Figure (7) represents the MMF spectrum for each machine. The sign of each component represents the rotation direction. The positive sign indicates that the harmonic rotates in the same direction as the rotor; negative sign indicates that the two rotate in opposite direction.

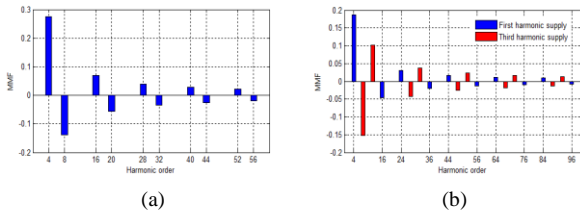


Fig. 7. MMF spectrum (a) 12/8/3 machine ; (b) 20/8/5 machine.

The model developed in [21] is used to estimate analytically the armature reaction PM losses. Figure (8) shows the analytical estimation and FE calculation of this losses. The 20/8/5 machine supplied by the first current harmonic has the lower armature reaction PM losses. Thus, this strategy of supply is used at high speed. When the 20/8/5 is supplied by only the third current harmonic, the losses are significant at high speed, which can lead to demagnetization. Thus, this strategy of control must be used only at low speed. The 20/8/5 has moderate losses when it is supplied by both the first and the third harmonic current, but this losses are still higher than those in 12/8/3 machine. Consequently, if the 20/8/5 machine is used in this application, it should be supplied by the first current harmonic at high speed, third current harmonic at low speed, both at intermediate speed. This conclusion is based on the armature reaction PM losses analysis. However, in order to determine the adequate strategy of supply, the total losses in PM should be calculated.

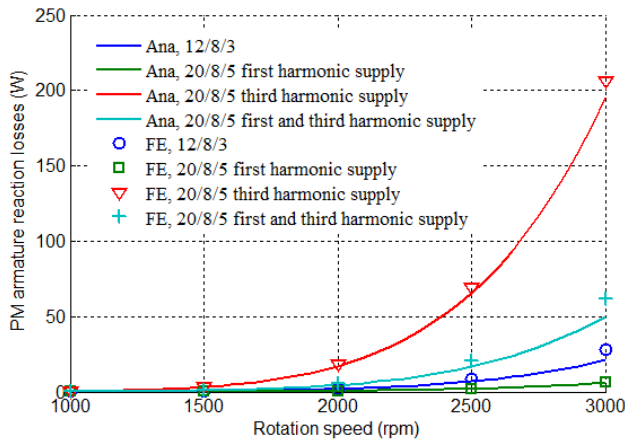


Fig. 8. Armature reaction PM losses.

Figure (9) shows the repartition of current density for the two machines. The following currents are due to armature reaction.

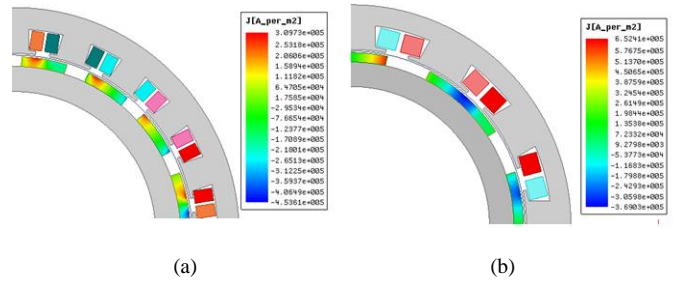


Fig. 9. Current density in PM. (a) 20/8/5 machine (first harmonic supply). (b) 12/8/3 machine.

### C. Load PM losses

In this section, the Total losses in PM are calculated when the two machines drive the mechanical load described by relation (1) and illustrated in figure (1). In these conditions, the total losses in PM are the combination of open circuit losses and armature reaction losses. Analytically, we estimate the PM losses by the sum of the open circuit losses and armature reaction losses. Numerically, the results are slightly different from the analytical sum as it can be observed in figure (10).

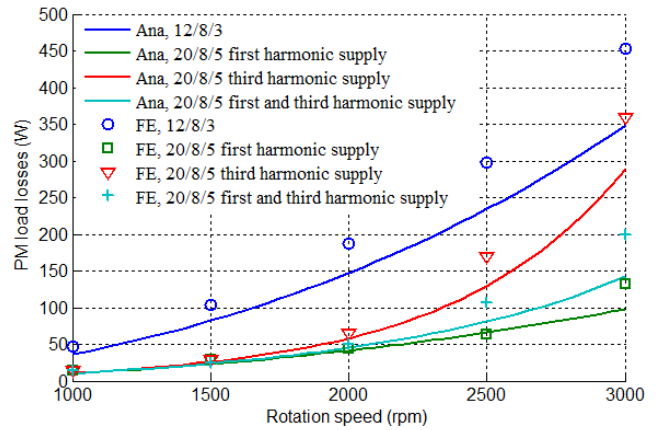


Fig. 10. PM losses on load in the two machines

The analytical and numerical estimations both indicate that the 20/8/5 machine has lower PM losses. This result is particularly appreciated because the evacuation of magnet losses to prevent from demagnetization is difficult to perform. More generally, in marine propulsion, the fault tolerant is appreciated, thus justifying the choice of the 20/8/5 machine to replace the 12/8/3 machine to drive the propeller. The supply strategy of 20/8/5 machine mentioned in (B) is confirmed. It is clear that the open circuit PM losses in 12/8/3 machine are large, which lead to significant PM losses in this machine.

## V. TOTAL LOSSES

Neglecting the iron losses, the overall losses in the two machines are the sum of copper losses and magnet losses. The results are shown in figure (11).

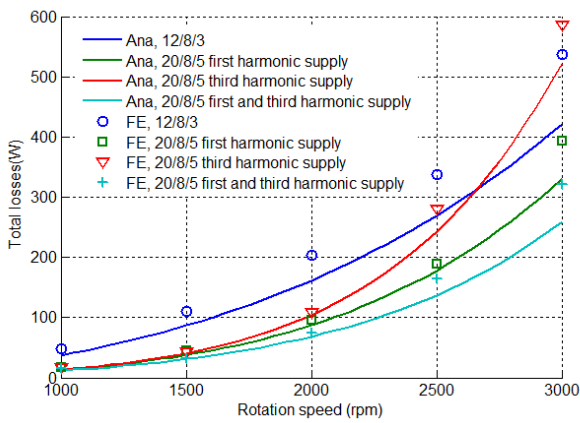


Fig. 11. Total losses in the two machines.

As we can see in figure (11), the 12/8/3 machine has more losses than the 20/8/5 machine over the whole speed range if this machine is supplied by the first harmonic or both the first and the third harmonic. Consequently, we have two benefits for replacing the 12/8/3 by 20/8/5 one:

- A reliable propeller is obtained since the five phase machine has a fault tolerant capability.
- Lower losses can be obtained if an appropriate supply is chosen. The first harmonic and the first and the third harmonic supply are the adequate supplies for 20/8/5 machine.

Despite the higher copper losses, we can see that 20/8/5 has lower magnet losses. This point is interesting because, these losses, unlike the copper losses, are difficult to be evacuated by cooling. Hence, the 20/8/5 machine can be considered as the reliable multiphase version of 12/8/3 machine.

## VI. CONCLUSION

In this paper, a comparison between two machines 20/8/5 and 12/8/3 with the same number 0.5 of slots per pole and per phase is presented. The comparison takes into account the different possible supply strategies which are available for the 20/8/5 machine for a given torque. It appears that, with the chosen configuration of rotor magnet, the injection of both first and third current harmonic is the best solution for the 20/8/5 studied machine. In this case, the total losses are lower than in 12/8/3 machine. Moreover, it appears that the magnet rotor losses which are more difficult to evacuate than stator losses are lower for the 20/8/5. So the 20/8/5 machine, even if it presents a low first harmonic winding factor, can be used favorably in fault-tolerant low voltage application as it is the case for marine propulsion application as pleasure boat. This machine should be supplied by the first current harmonic at high speed, and by the third current harmonic at low speed, in order to avoid important losses PM over the whole speed range.

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