Corrigendum

Corrigendum to "An extension of the polar method to the First-order Shear Deformation Theory of laminates" [Compos. Struct. 127 (2015) 328–339]

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The author regrets to inform that, after the publication of the paper, he found an error about the transformation law of Eq. (20) concerning the lamina transverse shear stiffness matrix when passing from the local material frame of the ply Γ to the global frame of the laminate Γ^{l}. Indeed, the components of this matrix behave like those of a second-rank symmetric plane tensor with the local frame turned clockwise by an angle δ_k around the x_3 axis. Therefore, the correct relationship is:

$$\begin{aligned}\widehat{Q}_{qq} &= T + R\cos 2(\Phi - \delta_k),\\ \widehat{Q}_{qr} &= \quad\; R\sin 2(\Phi - \delta_k),\\ \widehat{Q}_{rr} &= T - R\cos 2(\Phi - \delta_k).\end{aligned} \tag{20}$$

Unfortunately, this error sign causes a similar error in Eqs. (25), (26), (A.7) and (A.9) whose correct form is:

$$T_{H^*} = \begin{cases} T & \text{(basic)},\\ 2T & \text{(modified)},\end{cases}$$

$$R_{H^*}e^{i2\Phi_{H^*}} = \begin{cases} \frac{1}{n}Re^{i2\Phi}\sum_{k=1}^{n}e^{-i2\delta_k} & \text{(basic)},\\ \frac{1}{n^3}Re^{i2\Phi}\sum_{k=1}^{n}(3n^2-d_k)e^{-i2\delta_k} & \text{(modified)},\end{cases} \tag{25}$$

$$R_{H^*}e^{i2\Phi_{H^*}} = \begin{cases} R_{1A^*}\frac{R}{R_1}e^{i2(\Phi+\Phi_1-\Phi_{1A^*})} & \text{(basic)},\\ \frac{R}{R_1}e^{i2(\Phi+\Phi_1)}\left(3R_{1A^*}e^{-i2\Phi_{1A^*}} - R_{1D^*}e^{-i2\Phi_{1D^*}}\right) & \text{(modified)},\end{cases} \tag{26}$$

$$R_{H^*}e^{i2\Phi_{H^*}} = \begin{cases} \frac{1}{n}\sum_{k=1}^{n}[R\cos 2(\Phi-\delta_k) + iR\sin 2(\Phi-\delta_k)] & \text{(basic)},\\ \frac{1}{n^3}\sum_{k=1}^{n}(3n^2-d_k)[R\cos 2(\Phi-\delta_k) + iR\sin 2(\Phi-\delta_k)] & \text{(modified)},\end{cases} \tag{A.7}$$

$$R_{H^*}e^{i2\Phi_{H^*}} = \begin{cases} \frac{1}{n}Re^{i2\Phi}\sum_{k=1}^{n}e^{-i2\delta_k} & \text{(basic)},\\ \frac{1}{n^3}Re^{i2\Phi}\sum_{k=1}^{n}(3n^2-d_k)e^{-i2\delta_k} & \text{(modified)}.\end{cases} \tag{A.9}$$

On the other hand, this error gives rise to a modification of the proof (given in Appendix B) for obtaining the correct form of Eq. (26). The correct proof is given here below.

In order to analytically derive the link between the deviatoric part of matrix $[H^*]$ and the second anisotropic polar modulus R_1 and the related polar angle Φ_1 of matrices $[A^*]$ and $[D^*]$, let us consider the expression of quantities $\sum_{k=1}^{n}e^{-i2\delta_k}$ and $\sum_{k=1}^{n}(3n^2-d_k)e^{-i2\delta_k}$ appearing in Eq. (A.9). These quantities actually depend upon the polar parameters of the membrane and bending stiffness matrices of the laminate. To derive these relationships let us consider the following property of complex numbers:

$$\overline{\alpha z + \beta w} = \alpha\bar{z} + \beta\bar{w};\quad \text{with } z, w \in \mathbb{C} \quad \text{and } \alpha, \beta \in \mathbb{R}, \tag{B.0}$$

where $\bar{z}$ is the complex conjugate of z. By using property (B.0) and considering Eqs. (22) and (24) we have:

$$\begin{aligned}\sum_{k=1}^{n}e^{-i2\delta_k} &= \sum_{k=1}^{n}\overline{e^{i2\delta_k}} = \overline{\sum_{k=1}^{n}e^{i2\delta_k}} = n\frac{R_{1A^*}}{R_1}\overline{e^{i2(\Phi_{1A^*}-\Phi_1)}}\\ &= n\frac{R_{1A^*}}{R_1}e^{i2(\Phi_1-\Phi_{1A^*})},\end{aligned} \tag{B.1}$$

$$\begin{aligned}\sum_{k=1}^{n}d_k e^{-i2\delta_k} &= \sum_{k=1}^{n}d_k\overline{e^{i2\delta_k}} = \overline{\sum_{k=1}^{n}d_k e^{i2\delta_k}} = n^3\frac{R_{1D^*}}{R_1}\overline{e^{i2(\Phi_{1D^*}-\Phi_1)}}\\ &= n^3\frac{R_{1D^*}}{R_1}e^{i2(\Phi_1-\Phi_{1D^*})}.\end{aligned} \tag{B.2}$$

The expression of quantity $\sum_{k=1}^{n}(3n^2-d_k)e^{-i2\delta_k}$ can be obtained by combining Eqs. (B.1) and (B.2) as follows:

$$\begin{aligned}\sum_{k=1}^{n}(3n^2-d_k)e^{-i2\delta_k} &= 3n^2\sum_{k=1}^{n}e^{-i2\delta_k} - \sum_{k=1}^{n}d_k e^{-i2\delta_k}\\ &= \frac{n^3}{R_1}e^{i2\Phi_1}\left(3R_{1A^*}e^{-i2\Phi_{1A^*}} - R_{1D^*}e^{-i2\Phi_{1D^*}}\right).\end{aligned} \tag{B.3}$$

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Finally, by substituting Eqs. (B.1) and (B.3) into Eq. (A.9) (and after some standard passages) it is possible to obtain the desired result:

$$R_{H^*} e^{i2\Phi_{H^*}} = \begin{cases} \frac{1}{n} R e^{i2\Phi} n \frac{R_{1A^*}}{R_1} e^{i2(\Phi_1 - \Phi_{1A^*})} = R_{1A^*} \frac{R}{R_1} e^{i2(\Phi + \Phi_1 - \Phi_{1A^*})} & \text{(basic)}, \\ \frac{1}{n^3} R e^{i2\Phi} \frac{n^3}{R_1} e^{i2\Phi_1} \left(3R_{1A^*} e^{-i2\Phi_{1A^*}} - R_{1D^*} e^{-i2\Phi_{1D^*}}\right) & \\ \quad = \frac{R}{R_1} e^{i2(\Phi + \Phi_1)} \left(3R_{1A^*} e^{-i2\Phi_{1A^*}} - R_{1D^*} e^{-i2\Phi_{1D^*}}\right) & \text{(modified)}. \end{cases} \tag{B.4}$$

Nevertheless, the mentioned error does not affect at all nor the numerical results presented in the paper neither the conclusions, since the above relationships were correctly implemented into the numerical code employed to carry out all of the calculations.

The author would like to apologise for any inconvenience caused.